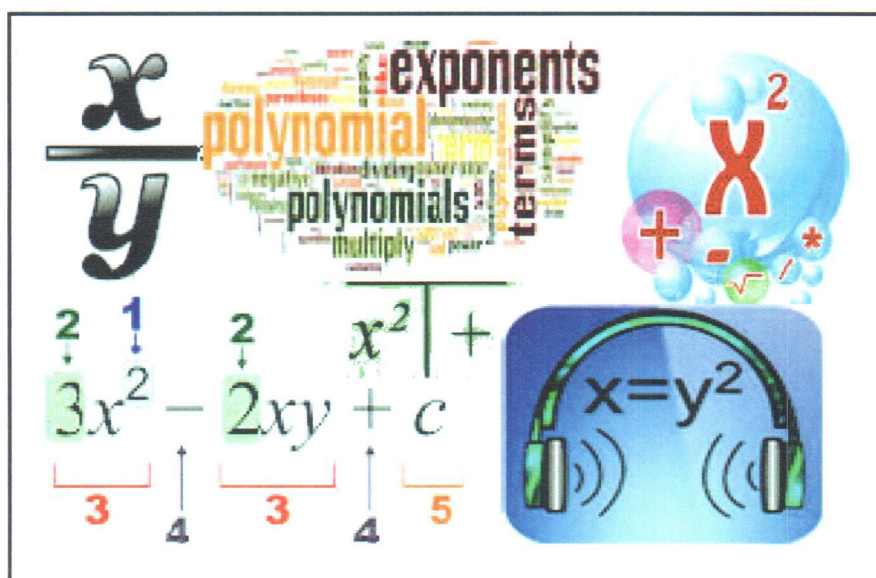


DEPARTMENT OF EDUCATION

GRADE 10

MATHEMATICS

UNIT 3



BASIC ALGEBRA

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PAPUA NEW GUINEA
2016

GRADE 10

MATHEMATICS

UNIT 3

BASIC ALGEBRA

**TOPIC 1: EXPANDING ALGEBRAIC
EXPRESSIONS**

**TOPIC 2: FACTORING ALGEBRAIC
EXPRESSIONS**

TOPIC 3: POWERS /INDICES

TOPIC 4: EQUATIONS

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MR. DEMAS TONGOGO
Principal-FODE



Flexible Open and Distance Education
Papua New Guinea

Published in 2016

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CONTENTS

	Page
Contents.....	3
Secretary's Message.....	4
Unit Introduction.....	5
Study Guide.....	6
TOPIC 1: EXPANDING ALGEBRAIC EXPRESSIONS.....	7
☐ Lesson 1: Collecting Like Terms.....	9
☐ Lesson 2: Expanding and Simplifying Algebraic Expressions.....	15
☐ Lesson 3: Multiplying Negative Terms.....	20
☐ Lesson 4: Expanding and Simplifying Expressions with Negative Terms.....	24
☐ Lesson 5: Expanding Binomial Products.....	30
☐ Lesson 6: Further Expansion of Binomial Products.....	37
Summary.....	42
Answers to Practice Exercises 1-6.....	43
TOPIC 2: FACTORING ALGEBRAIC EXPRESSIONS.....	47
☐ Lesson 7: Factorizing Simple Algebraic Expressions.....	49
☐ Lesson 8: Factorizing Quadratics: Common Monomial Factor.....	55
☐ Lesson 9: Factorizing Quadratics: Trinomials.....	58
☐ Lesson 10: Further Quadratics Factorization.....	64
☐ Lesson 11: Simplifying Quotients by Factorization.....	69
Summary.....	74
Answers to Practice Exercises 7-11.....	75
TOPIC 3: POWERS/INDICES.....	77
☐ Lesson 12: Positive Powers/Indices.....	79
☐ Lesson 13: Zero and Negative Powers.....	87
☐ Lesson 14: Fractional Indices.....	92
☐ Lesson 15: Scientific Notations.....	98
☐ Lesson 16: Addition and Subtraction of Numbers in Scientific Notation.....	108
☐ Lesson 17: Multiplication and Division of Numbers in Scientific Notation.....	112
Summary.....	118
Answers to Practice Exercises 12-17.....	120
TOPIC 4: EQUATIONS.....	123
☐ Lesson 18: Simple Equations.....	125
☐ Lesson 19: Equations Involving Grouping Symbols.....	136
☐ Lesson 20: Equations Involving Fractions.....	141
☐ Lesson 21: Problem Solving Involving Equations.....	148
☐ Lesson 22: Formulae and Substitution.....	155
☐ Lesson 23: Changing the Subject of a Formula.....	162
Summary.....	169
Answers to Practice Exercises 18-23.....	170
REFERENCES.....	173

SECRETARY'S MESSAGE

Achieving a better future by individuals students, their families, communities or the nation as a whole, depends on the curriculum and the way it is delivered.

This course is part and parcel of the new reformed curriculum – the Outcome Base Education (OBE). Its learning outcomes are student centred and written in terms that allow them to be demonstrated, assessed and measured.

It maintains the rationale, goals, aims and principles of the national OBE curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision of Flexible, Open and Distance Education as a alternative pathway of formal education.

The Course promotes Papua New Guinea values and beliefs which are found in our constitution, Government policies and reports. It is developed in line with the National Education Plan (2005 – 2014) and addresses an increase in the number of school leavers which has been coupled with a limited access to secondary and higher educational institutions.

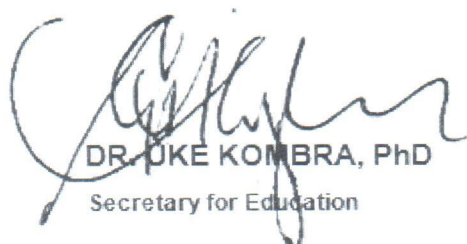
Flexible, Open and Distance Education is guided by the Department of Education's Mission which is fivefold;

- to facilitate and promote integral development of every individual
- to develop and encourage an education system which satisfies the requirements of Papua New Guinea and its people
- to establish, preserve, and improve standards of education throughout Papua New Guinea
- to make the benefits of such education available as widely as possible to all of the people
- to make education accessible to the physically, mentally and socially handicapped as well as to those who are educationally disadvantaged

The College is enhanced to provide alternative and comparable path ways for students and adults to complete their education, through one system, many path ways and same learning outcomes.

It is our vision that Papua New Guineans harness all appropriate and affordable technologies to pursue this program.

I commend all those teachers, curriculum writers and instructional designers, who have contributed so much in developing this course.



DR. OKE KOMBRA, PhD
Secretary for Education

UNIT 3: BASIC ALGEBRA

Introduction



Dear Student,

This is the Third Unit of Grade 10 Mathematics Course. It is based on the CDAD Lower Secondary Syllabus and Curriculum Framework for Grade 10

This Unit consists of four topics:

- | | |
|-----------------|--|
| Topic 1: | Expanding Algebraic Expressions |
| Topic 2: | Factoring Algebraic Expressions |
| Topic 3: | Powers/Indices |
| Topic 4: | Equations |

In **Topic 1 Expanding Algebraic Expressions**, you will revise how to simplify algebraic expressions by collecting like terms. You will also learn how to expand expressions which contain grouping symbols (brackets, parentheses and braces) then you will learn the meaning of a binomial product and finally learn how to expand and simplify a binomial product.

In **Topic 2 Factoring Algebraic Expressions**, you will learn the meaning of factorization and how to factorize simple algebraic expressions. You will also define and discuss quadratic expressions and give examples. Then you will learn to identify quadratic expressions and discover how to factorize quadratic expressions.

In **Topic 3 Powers/Indices**, you will learn how to revise the rules used to simplify positive exponents or powers and learn about negative powers. You will then learn how to write very large numbers and very small numbers in short form using scientific notation. Finally, you will find the square roots of numbers using square root tables.

In **Topic 4 Equations**, you will learn how to solve many types of equations such as those involving brackets and fractions. You will also learn to solve problems involving equations. Then you will learn how to substitute numerical values into formulas and change the subject of a formula or equation.

You will find that each lesson has reading materials to study, worked examples to help you, and a Practice Exercise. The answers to practice exercise are given at the end of each topic.

All the lessons are written in simple language with comic characters to guide you and many worked examples to help you. The practice exercises are graded to help you to learn the process of working out problems.

We hope you enjoy using this Unit.

Mathematics Department
FODE

STUDY GUIDE

Follow the steps given below as you work through the Unit.

Step 1: Start with Topic 1 Lesson 1 and work through it.

Step 2: When you complete Lesson 1, do Practice Exercise 1.

Step 3: After you have completed Practice Exercise 1, check your work. The answers are given at the end of TOPIC 1.

Step 4: Then, revise Lesson 1 and correct your mistakes, if any.

Step 5: When you have completed all these steps, tick the Lesson check-box on the Contents Page like this:

☒ Lesson 1: Collecting Like Terms

Then go on to the next Lesson. Repeat the process until you complete all of the lessons in Topic 1.

As you complete each Lesson, tick the check-box for that lesson, on the Contents Page, like this ☒. This helps you to check on your progress.

Step 6: Revise the Topic using Topic 1 Summary, then, do Topic Test 1 in Assignment 3.

Then go on to the next Topic. Repeat the process until you complete all of the four Topics in Unit 3.

Assignment: (Four Topic Tests and a Unit Test)

When you have revised each Topic using the Topic Summary, do the Topic Test in your Assignment Book. The Course Book tells you when to do each Topic Test.

When you have completed the four Topic Tests, revise well and do the Unit test. The Assignment Book tells you when to do the Unit Test.

The Topic Tests and the Unit Test in the Assignment will be marked by your Distance Teacher. The marks you score in each Assignment Book will count towards your final mark. If you score less than 50%, you will repeat that Assignment Book.

Remember, if you score less than 50% in three Assignments, you will not be allowed to continue. So, work carefully and make sure that you pass all of the Assignments.

TOPIC 1

EXPANDING ALGEBRAIC EXPRESSIONS

Lesson 1:	Collecting Like Terms
Lesson 2:	Expanding and Simplifying Algebraic Expressions
Lesson 3:	Multiplying Negative Terms
Lesson 4:	Expanding and Simplifying Expressions with Negative Terms
Lesson 5:	Expanding Binomial Products
Lesson 6:	Further Expansion of Binomial Products

TOPIC 1: EXPANDING ALGEBRAIC EXPRESSIONS

Introduction



Algebra is a branch of mathematics designed to help solve certain types of problems quicker and easier. It is based on the concept of unknown values called variables, unlike arithmetic which is based entirely on known number values.

Algebraic expression is a symbol or a combination of symbols used in Algebra and containing one or more numbers, variables and arithmetic operations.

Here are some examples of algebraic expressions:

1. $3a^2$
2. $2ab - 4c$
3. $x + y - z$
4. $6x^2y + 3xy + 5y$

In order for you to progress well in Mathematics, you need to be able to read and write algebraic expressions, and be skilled in the application of algebraic expressions.

In your study of Grades 7, 8 and 9 Mathematics, you have come across simple algebraic manipulations. In this Topic, you will learn further algebraic manipulations.

First, you will revise the meaning of “like or similar terms” and then look at how to simplify algebraic expressions involving like terms. You have learned these in your early study of Mathematics.

You will then look at how to expand or multiply algebraic expressions having positive terms and negative terms contained within grouping symbols (brackets $[]$, parentheses $()$ and braces $\{ \}$). You will learn how to simplify the resulting expansions by collecting like terms.

You will also learn about what binomial expression is. Finally, you will learn how to expand simple binomial products.

Lesson 1: Collecting Like Terms



Welcome to the first lesson of your Grade 10, Unit 3, Topic 1.



In this lesson, you will:

- revise the meaning of numerical and algebraic expressions
- define terms and identify like or similar terms
- simplify algebraic expressions by collecting like terms.

Let us revise some of the basic concepts we came across in our earlier study of algebra.

Expressions that contain only constants are called **numerical expressions**. It is a set of numerical values that are separated by the four mathematical operations addition (+), subtraction (−), multiplication (x) and division (÷).

Examples: (i) $7 + 9$ (ii) $9 + 8 - 5$

A **numerical term** or **constant term** is simply a number.

Examples: (i) 6 (ii) $\frac{2}{3}$ (iii) 14

An algebraic term is either a single variable (a letter that stands for a number). Or it can be a combination involving multiplication, division (or both) of a number and one or more variables.

Examples: (i) x^2 (ii) $3ab$ (iii) $\frac{2}{3}mn$

Remember that when multiplying, we do not use the sign of multiplication cross(x), instead we use a centered dot (•) because we do not want to confuse it with the letter x which is used as a variable to stand for numbers.

Examples:

- (i) $a \cdot b = ab$ (read as “a times b equals ab”)
 (ii) $3 \cdot n = 3n$ (read as “3 times n equals 3n”)
 (iii) $2 \cdot a \cdot b \cdot 3 \cdot c = 6abc$ (read as “2 times a times b times 3 times c equals 6abc”)

When a variable is multiplied by itself then we write this by using the “**squared notation**”; that is, writing the variable with an exponent of 2.

Examples: (i) $y \cdot y = y^2$
 (ii) $4 \cdot p \cdot p = 4p^2$

Generally, the variables in a term are written in alphabetical order.

Example: The product of $5c$ and $6d$ is written as **$30cd$** and not **$30dc$** .

The constants and the variables, whose product makes an algebraic term, are called **factors** of the term. The numerical factor of an algebraic term is called its **coefficient**. The variable factors of an algebraic term are called **algebraic factors**.

For example:

- (i) **$abcd$** is an algebraic term of four variable factors **a** , **b** , **c** , and **d** .
- (ii) **$4x$** is an algebraic term of two factors. The numerical factor **4** and the variable factor **x** . We say **4** is the **coefficient** of **x** .

Remember, that an algebraic term formed only by variable factors is understood to be having a coefficient of 1.

A combination of algebraic terms or a group of numerical term and algebraic terms which are added or subtracted from one another is called an **algebraic expression**.

Each of the following is an algebraic expression formed only by algebraic terms.

(i) $a + b$ (ii) $2xy - z + yz$ (iii) $\frac{3}{5}mn - 2p + q$

Each of the following algebraic expressions is formed from both algebraic and numerical terms.

(i) $y + 8$ (ii) $3uv - \frac{3}{4}$ (iii) $9 + \frac{3x}{y} - 5w$

Algebraic expressions that contain only one term are called **monomials**.

Examples: (i) x^2 (ii) $3ab$ (iii) $\frac{2}{3}mn$

Algebraic expressions that contain two terms are called **binomials**.

Examples: (i) $a + b$ (ii) $y + 8$ (ii) $2x^2 + 3y^2$

Algebraic expressions that contain three terms are called **trinomials**.

Examples: (i) $4xy - 2z + 3yz$ (ii) $\frac{3}{5}mn - 5p + 7q$ (iii) $x^2 + 5x + 6$

Algebraic expressions that have two or more terms are called **polynomials**.

Example: $x^4 + x^3y - x^2y^2 - xy^3 + y^4 5x + 6$

So binomials and trinomials are also polynomials.

In Algebra, we often get very long expressions that we need to make simpler. Simple expressions are easier to solve.

In this section, we will learn to simplify algebraic expression by combining or collecting like terms.

Combining or Collecting Like Terms is a process used to simplify algebraic expression using addition and subtraction of the coefficient of terms.

The key to using and understanding the method of collecting like terms is to understand like terms and be able to identify when pairs of terms are like terms. Some examples of like terms are given below:

- (i) $2x, 25x, x, -2x, -x$ because each term consist of a single variable x and a numerical coefficient
- (ii) $15, -4, 25, 1032, 0.6$ because they are all constants.
- (iii) $34y^2, -y^2, 5y^2, y^2$ because they have the same literal coefficient (y^2)

For comparison, below are examples of unlike terms.

- (i) $11x, 11y$ both have a single variable with an exponent of 1, but the terms are not alike since different variables are used.
- (ii) $15y, 20y^2, 21y^5$ these are unlike terms because each y variable in the terms has a different exponent.
- (iii) $9xy, 4x$ these are unlike terms because only one term has the y variable although both have an x variable.

Like terms are two or more terms that have exactly the same variable with the same exponent.

Now, let us look at some examples. We are going to simplify algebraic expressions by collecting like terms.

Example 1

Simplify: $5x + 3y + 4x + 2y$.

Solution: $5x + 3y + 4x + 2y$

$= (5x + 4x) + (3y + 2y)$ Collect like terms in parentheses

$= 9x + 5y$ Add the coefficient of like terms

Example 2

Simplify: $4r - 5s + 2rs - 8s - 3r$

Solution: $4r - 5s + 2rs - 8s - 3r$

$$= (4r - 3r) + 2rs + (-5s - 8s)$$

Collect like terms by parentheses using the rule of signs for addition and subtraction

$$= r + 2rs + (-13s)$$

$$= \mathbf{r + 2rs - 13s}$$

Add like terms

Example 3

Simplify: $-2y^2 + 4y - 6y^2 - 2y$

Solution: $-2y^2 + 4y - 6y^2 - 2y$

$$= (-2y^2 - 6y^2) + (4y - 2y)$$

Collect like terms by parentheses

$$= \mathbf{-8y^2 + 2y}$$

add like terms

Example 4

Simplify: $5x^2y + 2x^2y - 5xy - 3 + 2xy - 2$

Solution: $5x^2y + 2x^2y - 5xy - 3 + 2xy - 2$

$$= (5x^2y + 2x^2y) + (-5xy + 2xy) + (-3 - 2)$$

$$= 7x^2y + (-3xy) + (-5)$$

$$= \mathbf{7x^2y - 3xy - 5}$$

Example 5

Simplify: $8x^3y^2 + 3x^3 - 7x^2y^3 + 9x^3y^2$

Solution: $8x^3y^2 + 3x^3 - 7x^2y^3 + 9x^3y^2$

$$= (8x^3y^2 + 9x^3y^2) + 3x^3 - 7x^2y^3$$

$$= \mathbf{17x^3y^2 + 3x^3 - 7x^2y^3}$$

If an algebraic expression has no like terms, it cannot be simplified any further.

NOW GO TO PRACTICE EXERCISE 1

**Practice Exercise 1**

1. Express the following terms in their simplest form. Write the answer on the space provided.

(a) $6 \cdot p \cdot q \cdot y$ **Answer:** _____

(b) $5 \cdot s \cdot t$ **Answer:** _____

(c) $3 \cdot a \cdot a \cdot b$ **Answer:** _____

(d) $5 \cdot x \cdot y \cdot 4 \cdot x \cdot y \cdot 3$ **Answer:** _____

(e) $j \cdot 5 \cdot k \cdot 2 \cdot j \cdot 4$ **Answer:** _____

(f) $7 \cdot b \cdot b \cdot m \cdot m$ **Answer:** _____

(g) $5 \cdot 2 \cdot 3 \cdot f \cdot g \cdot f \cdot h \cdot h$ **Answer:** _____

(h) $2 \cdot 0 \cdot a \cdot a \cdot b$ **Answer:** _____

2. The expressions on the right have their like terms combined. Match the simplified expression in Column B with the expression on Column A. Write the letter of the correct expression on the space provided before each number.

COLUMN A

- _____ 1) $8x - 3x$
_____ 2) $3c + 9y - 5c$
_____ 3) $-4m + 5m - 7mn$
_____ 4) $6ab + 4bc - 3ab + bc$
_____ 5) $7x^2y - 2x^2y + 5xy^2 - 3xy^2$
_____ 6) $-4m - 7mn + 8n$
_____ 7) $8c + 9y - 5c$

COLUMN B

- a. $5x^2y + 2xy^2$
b. $5x$
c. $3c + 9y$
d. $3ab + 5bc$
e. $-2c + 9y$
f. $-4m - 7mn + 8n$
g. $m - 7mn$

3. Simplify each expression.

(a) $b - 3 + 6 - 2b$

(b) $2n - t - n + 4t$

(c) $abc + 4 - 5abc - 3 + 6$

(d) $-a^2b + 5 + ab^2 + 3$

(e) $-2ab + a + b + 4ab + a$

(f) $2a - 2b - 5c + 8$

(g) $p^2 - 3p - 10 + 2p^2 + p - 2$

(h) $x^2 + x - 6 + x^2 + 8x + 15$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.
--

Lesson 2: Expanding and Simplifying Algebraic Expressions



In the previous lesson, you have learned how to simplify algebraic expressions by collecting or combining like terms.



In this lesson, you will:

- identify grouping symbols such as brackets, braces and parentheses
- define the term expanding
- expand algebraic expressions having positive terms which are contained within the grouping symbols.

Let us look at the importance of using grouping symbols in expressions.

Grouping symbols are used to show which part of the expression should be done first.

There are basically three types of grouping symbols: Parentheses (), Brackets [], and Braces { }.

- Parentheses () are used to group numbers or variables. Everything inside parentheses must be done before any other operations.
- Brackets [] and Braces { } are also used to group numbers and variables. Technically, they are used after parentheses.

Parentheses are to be used first, then brackets, and then braces: { [()] }

Consider we had the statement “multiply 6 by the sum of x and y ”.

To show that the operation of adding x and y is done before multiplying by 6, we enclose $x + y$ in a “parentheses” ().

So the statement is written as the algebraic expression: $6(x + y)$

Note that if in writing the expression the “parentheses” () are left out and written as $6x + y$, this would mean as “multiply x by 6 and then add y ” which is not the same as the original statement.

Grouping symbols are the first step in the longer process of determining the order of operations which are taught in this lesson and the next.

Expanding Algebraic Expressions

We have come across the phrase “expanding algebraic expressions” in your Grade 8 and 9 Mathematics courses. It means that we rewrite expressions in an equivalent form that do not use grouping symbols. Hence

Expanding means removing the { }, [] or the () from the expressions.

For example, consider the example above that is:

$$6(x + y)$$

How can we write this in an equivalent expression without using parentheses? Meaning how do we expand the expression $6(x + y)$?

In Algebra, putting together next to each other usually means to multiply.

The expression $6(x + y)$ means “to multiply 6 by $(x + y)$ ”.

Hence $6(x + y) = 6x + 6y$

This is called the Distributive law, which is a rule that holds true in the arithmetic of numbers. In general

$$a(b + c) = ab + ac$$

Therefore, to expand algebraic expressions, **multiply each term inside the grouping symbol by the number or term immediately to the left of the grouping symbol.**

Example 1

Expand: $5(x + 2y)$

Solution: $5(x + 2y) = 5(x) + 5(2y) = 5x + 10y$

Example 2

Expand: $7(a + 2b) + 4a$

Solution: $7(a + 2b) + 4a = 7a + 14b + 4a$
 $= 7a + 4a + 14b$
 $= 11a + 14b$

There are two or more grouping symbols in an expression, expand each grouping symbol in turn and then simplify the result by collecting like terms (if there are any).

Example 3

Expand: $2(m + n) + 0.4(2m + 3n)$

Solution: $2(m + n) + 0.4(2m + 3n) = 2m + 2n + 0.8m + 1.2n$
 $= 2m + 0.8m + 2n + 1.2n$
 $= 2.8m + 3.2n$

If the answer is an expression that contains a numerical term, that is just a number (constant), then we normally rewrite the answer so that the numerical term is at the end of the expression. See Examples 4 and 5 on the next page.

Example 4 Expand: $14 + 6\left(\frac{1}{2}a + x\right)$

Solution: $14 + 6\left(\frac{1}{2}a + x\right) = 14 + 6\left(\frac{1}{2}a\right) + 6(x)$

$$= 14 + 3a + 6x$$

$$= 3a + 6x + 14$$

Example 5 Expand: $7 + 5d(d + 4e + 8f)$

Solution: $7 + 5d(d + 4e + 8f) = 7 + 5d^2 + 20de + 40df$

$$= 5d^2 + 20de + 40df + 7$$

NOW DO PRACTICE EXERCISE 2

**Practice exercise 2**

Expand the following expressions and simplify your expansions where possible.

1. $5(3a + 4)$

Answer: _____

2. $9(x + 12)$

Answer: _____

3. $5 + 4(m + 3)$

Answer: _____

4. $8(c + d + 4)$

Answer: _____

5. $1 + 7(x + 5y)$

Answer: _____

6. $5x + 4 + 8(x + y)$

Answer: _____

7. $6(a + 7b + 4c + 2)$

Answer: _____

8. $6x(x + 15)$

Answer: _____

9. $0.3(12x + 6y + 12)$

Answer: _____

10. $8(y^2 + 1) + 7y^2 + 3$

Answer: _____

11. $6 + 4(5x + 3) + 4(x + y)$

Answer: _____

12. $x(x + 5y) + 4xy(x + y + 6)$

Answer: _____

13. $4(2x + 5) + \frac{1}{2}x(6x + 4) + 8$

Answer: _____

14. $m(m + n + p + 3) + 4(3m + 7n + 9p)$

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 3: Multiplying Negative Terms



You have identified the basic types of grouping symbols and their uses in simplifying algebraic expressions in the previous lessons. You have also learned to expand and simplify algebraic expressions with positive terms.



In this lesson, you will:

- revise the sign rules for multiplication
- multiply negative terms using the multiplication sign rules.

Let us revise what negative numbers are.

Negative numbers are numbers which are preceded by a minus or negative sign. They are numbers whose values are less than zero.

For example: -4 , -12 , $-\frac{2}{3}$

You have already learned the rules in multiplying signed numbers in your Grades 8 and 9 Mathematics.

Here are the rules to remind you.

Multiplying same signs : The product is positive
Multiplying different signs : The product is negative.

Thus: $(+)(+) = +$ $(-)(-) = +$ $(+)(-) = -$ $(-)(+) = -$

Note that, if a number or answer is positive we can omit the (+) sign in front of the number.

For example: $(+5)(+8) = +40$ is written as $(5)(8) = 40$

Similarly for $(-4)(-6) = +24$ is written as $(-4)(-6) = 24$

Thus, a number or a term with no sign is understood to be positive.

Here are further examples of the use of multiplication sign rule.

1. $(-5)(-6) = 30$
2. $(7)(-3) = -21$
3. $(-4)(9) = -36$
4. $(8)(9) = 72$
5. $(-3)(-4) = 12$

The multiplication rule is also applied for negative algebraic terms.

A **negative algebraic term** is a term that has a negative sign in front of either a single variable or a combination (involving multiplication, division or both) of a number and one or more variables.

Below are examples of negative algebraic terms:

1. $-x$
2. $-3y^2$
3. $-abc$
4. $-5xyz$
5. $-\frac{f}{g}$

The sign rules for multiplying negative algebraic terms together are the same as the sign rules used for multiplying negative numbers.

Example 1 Simplify: $(a)(-b)$

Solution: $(a)(-b) = -ab$

Example 2 Simplify: $(-y)(-y)$

Solution: $(-y)(-y) = y^2$

If three terms are multiplied together, first work out the sign of the product of the first two terms and then multiply this sign by the third term.

Example 3 Simplify: $(-5a)(-2b)(3c)$

Solution: $(-5a)(-2b)(3c)$

Step 1: $(-5a)(-2b) = 10ab \longleftarrow (-)(-) = +$

Step 2: $(10ab)(3c) = 30abc \longleftarrow (+)(+) = +$

Therefore the answer is $30abc$.

Example 4 Simplify: $(-2)(-6x)(-3y)$

Solution: $(-2)(-6x)(-3y)$

Step 1: $(-2)(-6x) = +12x \longleftarrow (-)(-) = +$

Step 2: $12x(-3y) = -36xy \longleftarrow (+)(-) = -$

Therefore the answer is $-36xy$.

If four terms are multiplied together, first work out the sign of the product of the first three terms and then multiply this sign by the fourth term.

5 Simplify: $(3a)(-b)(4c)(-4d)$

Solution: $(3a)(-b)(4c)(-4d)$

Step 1: $(3a)(-b) = -3ab$

Step 2: $(-3ab)(4c) = -12abc$

Step 3: $(-12abc)(-4d) = 48abcd$

Therefore the answer is 48abcd.

We extend the above process to find the final sign when any number of algebraic terms is multiplied together.

For example, if five terms are multiplied together, first, work out the sign of the product of the first four terms and then multiply this sign by the fifth term.

Note: When setting out your solution, there is no need to show the steps since you can probably work out the final answer in your mind straight away, nor is there a need to write the notes.

Here are some more examples.

1. Simplify: $(-3)(-11)$

Solution: $(-3)(-11) = 33$

Answer: 33

2. Simplify: $(-5)(-7h)$

Solution: $(-5)(-7h) = 35h$

Answer: 35h

3. Simplify: $(9h)(-6i)$

Solution: $(9h)(-6i) = -54hi$

Answer: -54hi

4. Simplify: $(-6ab)(-4ac)$

Solution: $(-6ab)(-4ac) = 24a^2bc$

Answer: $24a^2bc$

5. Simplify: $(2p)(-4q)(r^2)$

Solution: $(2p)(-4q)(r^2) = -8pqr^2$

Answer: $-8pqr^2$

6. Simplify: $(-k)(-6m)(-8km)(n)$

Solution: $(-k)(-6m)(-8km)(n) = -48k^2m^2n$

Answer: $-48k^2m^2n$

NOW DO PRACTICE EXERCISE 3

**Practice Exercise 3**

Multiply the following leaving your answer in its simplest form:

1. $(-4)(-9)$

Answer: _____

2. $(7)(-6)$

Answer: _____

3. $(-9)(-m)$

Answer: _____

4. $(8q)(-q)$

Answer: _____

5. $(4r)(8r)(-s)$

Answer: _____

6. $(-7b)(-6b)(-c)$

Answer: _____

7. $(-w)(-x)(-y)(-z)$

Answer: _____

8. $(-5a)(-6a)(b)(3c)$

Answer: _____

9. $(9n)(-6mn)(-6p)$

Answer: _____

10. $(2a^2)(-2b)(c)(-d)(-3e)$

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 4: Expanding and Simplifying Expressions with Negative Terms



You have defined negative terms and revised the rules for multiplying positive and negative numbers in the previous lesson. You also learned to expand negative terms by applying the rules of signs.



In this lesson, you will:

- expand and simplify algebraic expressions with negative terms.
-

Expanding expressions involves removing the grouping symbols from an expression by multiplying out the grouping symbols. This is achieved by multiplying every term inside the grouping symbols by the term outside the grouping symbols.

Also when collecting like terms after expanding the grouping symbols, it is often necessary to use the sign rules for addition of negative numbers or terms.

Below are examples to remind you of the sign rules which you dealt with in Lesson 1.

Example 1

$$-4 + -5 = -(4 + 5) = -9$$

Note: $-4 - 5$ is the same as $-4 + -5$, so $-4 - 5 = -9$

Example 2

$$3x + -7x = -4x$$

Note: $3x - 7x$ is the same as $3x + -7x$, so $3x - 7x = -4x$

Remember, when expanding a parentheses or a bracket, we multiply every term inside the parentheses or bracket by the term outside immediately to the left of the parentheses or bracket.

Here are the examples.

Example 1 Expand: $-4(x + 2)$

Solution: $-4(x + 2) = -4(x) + -4(2)$
 $\quad\quad\quad = -4x - 8$

Example 2 Expand: $-0.6(y - 8)$

Solution: $-0.6(y - 8) = -0.6(y) + -0.6(-8)$
 $\quad\quad\quad = -0.6y + 4.8$

Now, here is a third example where the term in front of the bracket or parentheses is only a minus or negative sign.

Example 3 Expand: $-(4b + 6c)$

Solution: $-(4b + 6c)$ is also the same as $-1(4b + 6c)$ since multiplying by 1 does not change the value of the original expression.

$$\begin{aligned}\text{Hence } -(4b + 6c) &= -1(4b + 6c) \\ &= -1(4b) + -1(6c) \\ &= \mathbf{-4b - 6c}\end{aligned}$$

Example 4 Expand: $-(3x - 5y)$

Solution: $-(3x - 5y)$ is also the same as $-1(3x - 5y)$ since multiplying by 1 does not change the value of the original expression.

$$\begin{aligned}\text{Hence } -(3x - 5y) &= -1(3x - 5y) \\ &= -1(3x) + -1(-5y) \\ &= \mathbf{-3x + 5y}\end{aligned}$$

After expanding the parentheses or brackets, we then simplify and collect like terms. Below are examples showing this method.

Example 5 Expand and simplify: $4(3 - 4a) - 5(6 + a)$

$$\begin{aligned}\text{Solution: } 4(3 - 4a) - 5(6 + a) &= 4(3) + 4(-4a) + -5(6) + -5(a) \\ &= 12 - 16a - 30 - 5a \\ &= 12 - 30 - 16a - 5a \quad \leftarrow \text{Collect like terms} \\ &= \mathbf{-18 - 21a}\end{aligned}$$

Example 6 Expand and simplify: $-3x(5x - y + z) + 4-(xy + 7)$

$$\begin{aligned}\text{Solution: } -3x(5x - y + z) + 4-(xy + 7) &= -3x(5x - y + z) + 4 - 1(xy + 7) \quad \leftarrow \begin{array}{l} \text{Write the number 1 in front} \\ \text{of the last parentheses for} \\ \text{easier multiplication.} \end{array} \\ &= -3x(5x) + -3x(-y) + -3x(z) + 4 + -1(xy) + -1(7) \\ &= -15x^2 + 3xy - 3xz + 4 - xy - 7 \\ &= -15x^2 + 3xy - xy - 3xz + 4 - 7 \quad \leftarrow \text{Collect like terms} \\ &= \mathbf{-15x^2 + 2xy - 3xz - 3}\end{aligned}$$

Example 7 Expand and simplify: $5\{[4(y - 4) + 15] - [2(5y - 3) + 1]\}$

Solution: You are given an expression that contains parentheses (), brackets [] and braces { }. All these are grouping symbols that are used to perform the operations within them ahead of other operations.

On the next page, are the steps to be done to solve the expression in the example 7.

Here again is the expression $5\{[4(y - 4) + 15] - [2(5y - 3) + 1]\}$.

Step 1: Remove the innermost grouping symbols which is the parentheses.

$$5\{[4(y - 4) + 15] - [2(5y - 3) + 1]\} \leftarrow \begin{array}{l} \text{Distributive law} \\ 4(y - 4) = 4y - 16 \\ 2(5y - 3) = 10y - 6 \end{array}$$

$$= 5\{[4y - 16 + 15] - [10y - 6 + 1]\}$$

Step 2: Now the expression contains only brackets and braces. Perform all the operations within the two sets of brackets.

$$5\{[4y - 16 + 15] - [10y - 6 + 1]\} \leftarrow \begin{array}{l} \text{Collect like terms} \\ -16 + 15 = -1 \\ -6 + 1 = -5 \end{array}$$

$$= 5\{[4y - 1] - [10y - 5]\}$$

Step 3: Now the two sets of brackets can be removed. However, the negative sign before the second set means that the terms within the brackets have to be multiplied by -1.

$$5\{[4y - 1] - [10y - 5]\} \leftarrow \begin{array}{l} -(10y - 5) = -1(10y - 5) \\ = -10y + 5 \end{array}$$

$$= 5\{4y - 1 - 10y + 5\}$$

Step 4: Now the expression contains only braces. Apply the distributive law to remove the braces. Then collect like terms.

$$5\{4y - 1 - 10y + 5\} \leftarrow \begin{array}{l} \text{Distributive law} \\ 5(4y) + 5(-1) + 5(-10y) + 5(5) \\ = 20y - 5 - 50y + 25 \end{array}$$

$$= 20y - 5 - 50y + 25 \leftarrow \begin{array}{l} \text{Collect like terms} \\ 20y + -50y = -30y \\ -5 + 25 = 20 \end{array}$$

$$= -30y + 20$$

Therefore, the simplified form of the expression $5\{[4(y - 4) + 15] - [2(5y - 3) + 1]\}$ is **$-30y + 20$**

Now look at the next example.

Example 8 Expand and simplify: $2x - \{3y + [-8 - 5y - (x - 4)]\}$

Solution:	$2x - \{3y + [-8 - 5y - (x - 4)]\}$	
	$= 2x - \{3y + [-8 - 5y - x + 4]\}$	remove parentheses
	$= 2x - \{3y - 8 - 5y - x + 4\}$	remove brackets
	$= 2x - 3y + 8 + 5y + x - 4$	remove braces
	$= 2x + x - 3y + 5y + 8 - 4$	collect like terms
	$= 3x + 2y + 4$	Answer

To sum up the work we have done:

When expanding and simplifying algebraic expressions,

- If the grouping symbol is preceded by a plus (+) sign, remove the grouping symbol and combine or collect like terms where necessary.
- If the grouping symbol is preceded by a negative sign remove the grouping symbol and change the signs of the terms inside the bracket or parentheses to their opposite (positive to negative or negative to positive), then collect or combine like terms where necessary.
- If the expression is involving a series of grouping symbols, start by removing from the innermost grouping symbol and apply the first and second bullets points.

Here are some more worked examples.

Example 1 Expand and simplify: $6(-2a - 3b)$

$$\begin{aligned}\text{Solution: } 6(-2a - 3b) &= 6(-2a) + 6(-3b) \\ &= -12a - 18b\end{aligned}$$

Answer: $-12a - 18b$

Example 2 Expand and simplify: $3c(c + 2d - 5e)$

$$\begin{aligned}\text{Solution: } 3c(c + 2d - 5e) &= 3c(c) + 3c(2d) + 3c(-5e) \\ &= 3c^2 + 6cd - 15ce\end{aligned}$$

Answer: $3c^2 + 6cd - 15ce$

Example 3 Expand and simplify: $7 + 4\left(\frac{1}{2}a + b\right)$

$$\begin{aligned}\text{Solution: } 7 + 4\left(\frac{1}{2}a + b\right) &= 7 + 4\left(\frac{1}{2}a\right) + 4(b) \\ &= 7 + 2a + 4b\end{aligned}$$

Answer: $7 + 2a + 4b$

NOW DO PRACTICE EXERCISE 4

**Practice Exercise 4**

1. Expand the following by removing the parentheses or brackets.

(a) $6(x + 3)$ Answer: _____

(b) $-3(m - n)$ Answer: _____

(c) $-x(x + 2)$ Answer: _____

(d) $2(3x - 4)$ Answer: _____

(e) $-3(x + y)$ Answer: _____

(f) $5(m - 4)$ Answer: _____

(g) $x(x + y)$ Answer: _____

(h) $y(y^2 - 2)$ Answer: _____

(i) $t(3t + 4)$ Answer: _____

(j) $4(y - 5)$ Answer: _____

2. Expand the following expression and simplify, if possible.

(a) $2(2p + 2q) + 2(3p - 3q)$ Answer: _____

(b) $3(2d + 3e) + 3(3d + 4e)$ Answer: _____

(c) $2(2e + 4f) - 4(3e + 3f)$ Answer: _____

(d) $3(4x + 2y) - 3(3x + 4y)$ Answer: _____

(e) $3(3t + 2u) + 3(4t - 3u)$ Answer: _____

(f) $3(6v + 4w) - 3(4v - 5w)$

Answer: _____

(g) $5(3q + 4r) + 3(2q + 4r)$

Answer: _____

(h) $m(m - 4) + 2(m + 1)$

Answer: _____

(i) $-4(x - 3) + 3(x + 2)$

Answer: _____

(j) $7(m - 3) - 2(m - 4)$

Answer: _____

3. Expand and simplify, where possible, by collecting like terms.

a) $x - y + [-12 - 6y - (x - 5)]$

Answer: _____

b) $\{4y + 8 - [6y - (x + 2)]\} + 4x$

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 5: Expanding Binomial Products



You have learned to expand and simplify algebraic expressions with negative terms in the previous lesson. You also learned to expand expressions containing a series of grouping symbols.



In this lesson, you will:

- define binomial expression and binomial product and give examples
- expand binomial products.

Let us define what a binomial expression is.

A **binomial expression** is an expression consisting of two terms.

For example the following are binomial expressions.

(a) $x + 7$ (b) $2x + 3$ (c) $6u - 5$ (d) $-5u - 7v$

Expanding Binomial Products

Let us define a binomial product first.

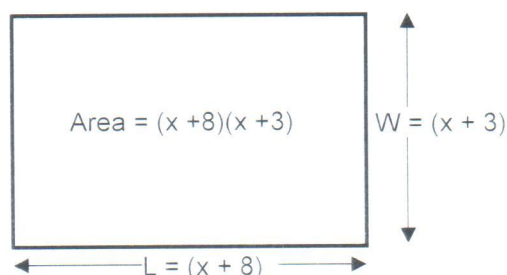
The product of two binomial expressions is called a **binomial product**.

For example, $(x + 3)(x + 8)$ is a binomial product.

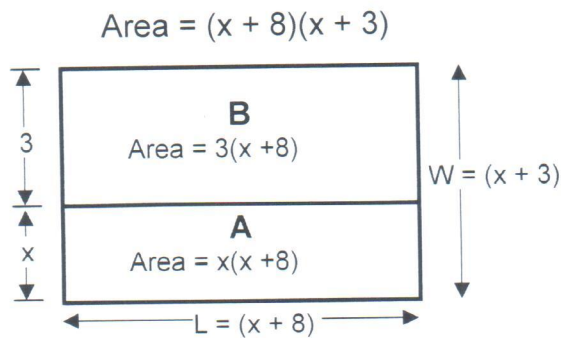
How do we expand a binomial product?

A binomial product is expanded by multiplying the two binomial expressions together and then collecting like terms.

The expansion of the binomial product $(x + 3)(x + 8)$ can be considered as a representative of the area of a rectangle of length $(x + 8)$ units and width $(x + 3)$ units.



This can be divided or split into two similar rectangles **A** and **B** shown on the next page.

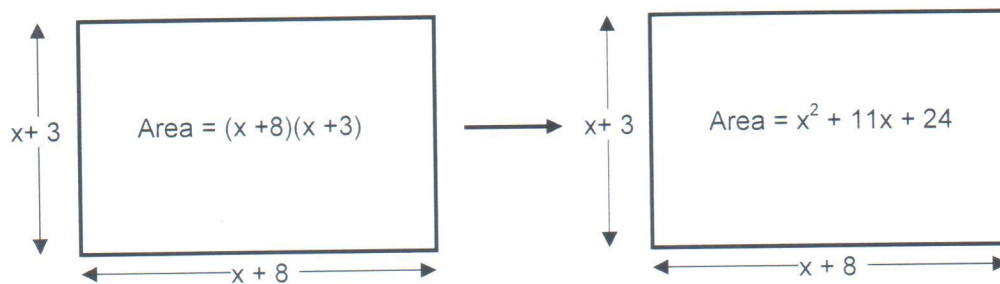


Clearly, Area of Original rectangle = Area of rectangle **A** + Area of rectangle **B**

$$\begin{aligned}
 (x + 8)(x + 3) &= x(x + 8) + 3(x + 8) \\
 &= x^2 + 8x + 3x + 24 \\
 &= x^2 + 11x + 24
 \end{aligned}$$

Collect like terms
8x and 3x

This result is shown in the diagrams below.



So the expansion of $(x + 8)(x + 3)$ is given by :

$$(x + 8)(x + 3) = x^2 + 11x + 24$$

This is the method we use to work the expansion of binomial products, but we do not need to draw the rectangular diagrams as in the above example. These were drawn in this particular instance to help explain the method.

Next is a shorter version which is called “**direct expansion method**” used to expand and simplify $(x + 8)(x + 3)$ and this is the method we will use from now on.

We multiply the second binomial $(x + 3)$ by the two terms of the first binomial $(x + 8)$.

That is, we multiply $(x + 3)$ by the first term x of the first binomial and then multiply $(x + 3)$ by the second term 8 of the first binomial. Then we collect like terms. So we have:

$$\begin{aligned}
 (x + 8)(x + 3) &= x(x + 3) + 8(x + 3) && \text{Multiply the second binomial by the two terms of the first binomial} \\
 &= x(x) + x(3) + 8(x) + 8(3) && \text{Expand brackets. Use Distributive property} \\
 &= x^2 + 3x + 8x + 24 && \text{Collect like terms. } 3x + 8x = 11x \\
 &= x^2 + 11x + 24 && \text{simplified form}
 \end{aligned}$$

Therefore, $(x + 8)(x + 3) = x^2 + 11x + 24$.

Example 2

Expand and then simplify $(x - 8)(x - 9)$.

Solution: $(x - 8)(x - 9) = x(x - 9) - 8(x - 9)$ Multiply the second binomial by the two terms of the first binomial

$$= x(x) + x(-9) + -8(x) + -8(-9)$$

Expand brackets.

$$= x^2 - 9x - 8x + 72$$

Collect like terms. $-9x - 8x = -17x$

$$= x^2 - 17x + 72$$

Simplified form

Therefore, $(x - 8)(x - 9) = x^2 - 17x + 72$.

Example 3

Expand and simplify the expression: $(2a - 3)(3a + 7)$

Solution: $(2a - 3)(3a + 7) = 2a(3a + 7) - 3(3a + 7)$ Multiply the second binomial by the two terms of the first binomial

$$= 2a(3a) + 2a(7) + -3(3a) + -3(+7)$$

Expand brackets.

$$= 6a^2 + 14a - 9a - 21$$

Collect like terms. $14a - 9a = 5a$

$$= 6a^2 + 5a - 21$$

Simplified form

Therefore, $(2a - 3)(3a + 7) = 6a^2 + 5a - 21$.

Now here is another example: Expand and then simplify $(m + n)(m - n)$.

Solution: $(m + 3)(m - 3) = m(m - 3) + 3(m - 3)$

$$= m(m) + m(-3) + 3(m) + 3(-3)$$

$$= m^2 - 3m + 3m - 9$$

$$= m^2 - 9$$

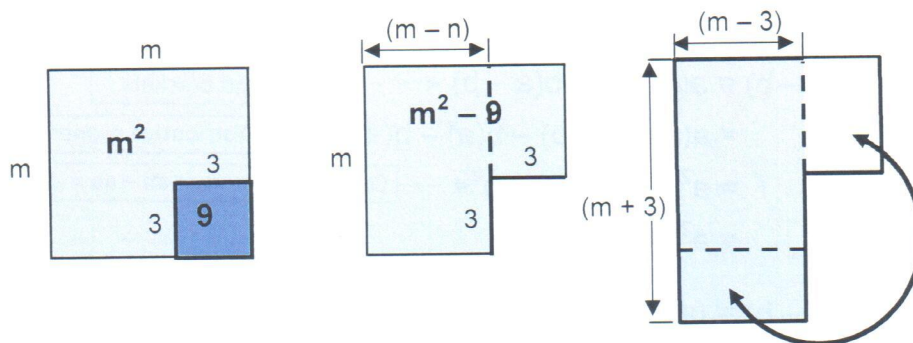
Therefore, $(m + 3)(m - 3) = m^2 - 9$.

Notice that in the above example, we are finding the product of the sum $(m + 3)$ and the difference $(m - 3)$ of two terms. The answer has only two terms, the square of the first term m^2 minus the square of the second term ($3^2 = 9$).

That was interesting! It ended up very simple.

And it is called the "difference of two squares". The two squares are m^2 and 9.

The illustration on the next page shows why it works.



$$m^2 - 9 \text{ is equal to } (m + 3)(m - 3)$$

Note: $(m - 3)$ could also be written first and $(m + 3)$ second:

$$(m - 3)(m + 3) = m^2 - 9$$

We can generalise this result to give us a rule:

The product of the sum and difference of two terms is equal to the square of the first term minus the square of the second term.

For example:

1. $(x + 5)(x - 5) = x^2 - 25$
2. $(b - 8)(b + 8) = b^2 - 64$
3. $(7 - y)(7 + y) = 49 - y^2$
4. $(a + 0.4)(a - 0.4) = a^2 - 0.16$
5. $(2b - 9)(2b + 9) = 4b^2 - 81$

Now, what will happen when we square a binomial (in other words, multiply it by itself)?

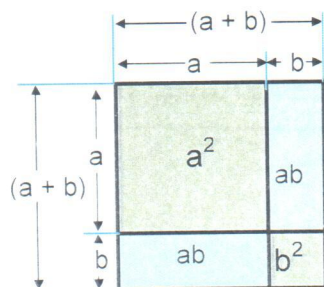
For example $(a + b)(a + b) = a(a + b) + b(a + b)$ ← Expand brackets.

$$= a(a) + a(b) + b(a) + b(b)$$
 ← Distributive property

$$= a^2 + ab + ab + b^2$$
 ← Collect like terms. $ab + ab = 2ab$

$$= a^2 + 2ab + b^2$$

The illustration shows why it works:



$$(a + b)(a + b) = a^2 + 2ab + b^2$$

$$(a + b)^2 = a^2 + 2ab + b^2$$

The same happens when you multiply $(a - b)$ by itself.

$$\begin{aligned}
 \text{For example: } (a - b)(a - b) &= a(a - b) - b(a - b) \quad \leftarrow \text{Expand brackets.} \\
 &= a(a) + -a(b) - b(a) - b(-b) \quad \leftarrow \text{Distributive property} \\
 &= a^2 - ab - ab + b^2 \quad \leftarrow \text{Collect like terms. } -ab - ab = -2ab \\
 &= a^2 - 2ab + b^2
 \end{aligned}$$

Hence, $(a - b)(a - b) = (a - b)(a - b)$

$$(a - b)^2 = a^2 - 2ab + b^2$$

We can generalise these results to give us a rule:

The square of a binomial is the sum of the square of the first terms, plus or minus twice the product of the two terms, and the square of the last term.

Here are some examples.

$$1. \quad (y + 1)^2 = y^2 + 2y + 1$$

Solution: The first term is y . Square it we have y^2
 The last term is 1 . Square it we have 1 .
 The product of y and 1 is y . Twice y is $2y$.
 Using the rule we have, $y^2 + 2y + 1$.

$$2. \quad (3x - 2)^2 = 9x^2 - 12x + 4$$

Solution: The first term is $3x$. Square it we have $9x^2$
 The last term is -2 . Square it we have 4 .
 The product of $3x$ and -2 is $-6x$. Twice y is $-12x$.
 Using the rule we have, $9x^2 - 12x + 4$.

$$3. \quad (4m + 3)^2 = 16m^2 + 24m + 9$$

$$4. \quad (2a - 3b)^2 = 4a^2 - 12ab + 9b^2$$

$$5. \quad (3x + 7)^2 = 9x^2 + 42x + 49$$

NOW DO PRACTICE EXERCISE 5

**Practice Exercise 5**

Expand and simplify the following.

(a) $(x + 5)(x + 6)$

Answer: _____

(b) $(3x + 6)(x + 2)$

Answer: _____

(c) $(7x + 3)(3x + 5)$

Answer: _____

(d) $(y + 6)(y - 1)$

Answer: _____

(e) $(3a - 5)(4a + 2)$

Answer: _____

(f) $(w - 2)(w - 3)$

Answer: _____

(g) $(6x + 4)\left(\frac{1}{2}x + 3\right)$

Answer: _____

(h) $(5x + 3)^2$

Answer: _____

(i) $(8b - 5)^2$

Answer: _____

(j) $(-2y - 5)(6y - 7)$

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.
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Lesson 6: Further Expansion of Binomial Products



You have learned to expand three special binomial products in the previous lesson.



In this lesson, you will:

- expand and simplify binomial products that have two variables.

In lesson 5, we limited our expansions to binomials that contained only one variable (letter). For example; $(x + 3)(x + 2)$ contains only one variable, x , and $(2y + 4)(y - 5)$ contains only one variable, y .

We will now look at the expansion of binomials that contain two different variables.

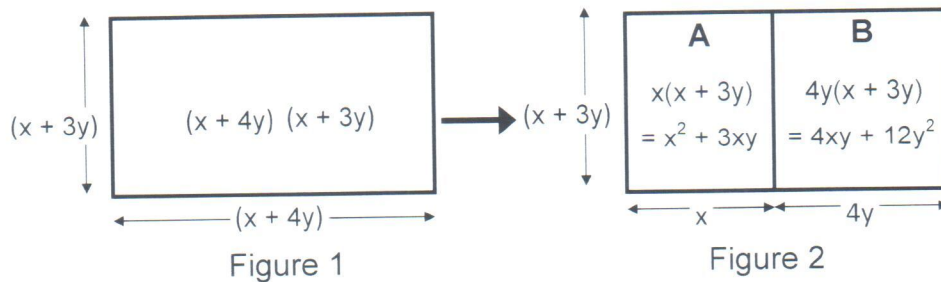
For example; $(x + 4y)(x + 3y)$ contains two variables, x and y .

How do we expand the binomial product $(x + 4y)(x + 3y)$?

Again let us consider a rectangle of length $(x + 4y)$ and breadth $(x + 3y)$ as in Lesson 5 and also see Figure 1 below.

$$\begin{aligned}\text{Area of rectangle} &= (\text{length})(\text{width}) \\ &= (x + 4y)(x + 3y)\end{aligned}$$

Now divide this rectangle into two smaller rectangles A and B (see Figure 2).



Now let us work out the areas of the two smaller triangles (**A** and **B**).

$$\begin{aligned}\text{Area of rectangle A} &= (\text{length})(\text{width}) \\ &= x(x + 3y) \\ &= x^2 + 3xy\end{aligned}$$

$$\begin{aligned}\text{Area of rectangle B} &= (\text{length})(\text{width}) \\ &= 4y(x + 3y) \\ &= 4xy + 12y^2\end{aligned}$$

Area of original rectangle = area of rectangle A + area of rectangle B

$$\begin{aligned}(x + 4y)(x + 3y) &= x^2 + 3xy + 4xy + 12y^2 \\ &= x^2 + 7xy + 12y^2\end{aligned}$$

So the expansion is given by: $(x + 4y)(x + 3y) = x^2 + 7xy + 12y^2$

We will use the method shown on the previous page to work out expansion of binomial products, but we do not need to draw and show the rectangles in the diagram.

As in Lesson 5, we can use the “**direct expansion method**” to expand and simplify binomials with two variables.

Below is the rule:

To expand and simplify two binomials with two variables, multiply the two terms of the second binomial by the two terms in the first binomial,

Example 1

Expand and then simplify: $(4x + y)(x + y)$

$$\begin{aligned}
 \text{Solution: } (4x + y)(x + y) &= 4x(x + y) + y(x + y) \leftarrow \text{Multiply 2}^{\text{nd}} \text{ binomial by the 1}^{\text{st}} \text{ binomial} \\
 &= 4x(x) + 4x(y) + y(x) + y(y) \leftarrow \text{Distributive property} \\
 &= 4x^2 + 4xy + xy + y^2 \leftarrow \text{Expand brackets} \\
 &= 4x^2 + 5xy + y^2 \leftarrow \text{Collect like terms: } (4xy + xy = 5xy)
 \end{aligned}$$

Therefore, $(4x + y)(x + y) = 4x^2 + 5xy + y^2$.

Example 2

Expand and then simplify: $(5a - 8b)(2a - 6b)$

$$\begin{aligned}
 \text{Solution: } (5a - 8b)(2a - 6b) &= 5a(2a - 6b) - 8b(2a - 6b) \leftarrow \text{Multiply the 2}^{\text{nd}} \text{ binomial by the 1}^{\text{st}} \text{ binomial} \\
 &= 5a(2a) + 5a(-6b) - 8b(2a) - 8b(-6b) \leftarrow \text{Distributive property} \\
 &= 10a^2 - 30ab - 16ab + 48b^2 \leftarrow \text{Expand brackets} \\
 &= 10a^2 - 46ab + 48b^2 \leftarrow \text{Collect like terms: } (-30ab - 16ab = -46ab)
 \end{aligned}$$

Therefore, $(5a - 8b)(2a - 6b) = 10a^2 - 46ab + 48b^2$.

Example 3

Expand and then simplify $(2a + 3b)(2a - 3b)$

$$\begin{aligned}
 \text{Solution: } (2a + 3b)(2a - 3b) &= 2a(2a - 3b) + 3b(2a - 3b) \\
 &= 2a(2a) + (2a)(-3b) + 3b(2a) + 3b(-3b) \\
 &= 4a^2 - 6ab + 6ab - 9b^2 \\
 &= 4a^2 + 0 - 9b^2 \\
 &= 4a^2 - 9b^2
 \end{aligned}$$

Therefore, $(2a + 3b)(2a - 3b) = 4a^2 - 9b^2$

Example 4

Expand and then simplify: $(2x + 10y)(\frac{1}{2}x + y)$

$$\begin{aligned}\text{Solution: } (2x + 10y)(\frac{1}{2}x + y) &= 2x(\frac{1}{2}x + y) + 10y(\frac{1}{2}x + y) \\ &= 2x(\frac{1}{2}x) + 2x(y) + 10y(\frac{1}{2}x) + 10y(y) \\ &= x^2 + 2xy + 5xy + 10y^2 \\ &= x^2 + 7xy + 10y^2\end{aligned}$$

$$\text{Therefore, } (2x + 10y)(\frac{1}{2}x + y) = x^2 + 7xy + 10y^2.$$

Example 5

Expand and simplify: $(2m + 3n)(2m - 3n)$

$$\begin{aligned}\text{Solution: } (2m + 3n)(2m - 3n) &= 2m(2m - 3n) + 3n(2m - 3n) \\ &= 2m(2m) + 2m(-3n) + 3n(2m) + 3n(-3n) \\ &= 4m^2 - 6mn + 6mn - 9n^2 \\ &= 4m^2 - 9n^2\end{aligned}$$

Note: In this example, we have found the product of the sum and difference of two terms. The sum of which is $(2m + 3n)$ and the difference is $(2m - 3n)$.

So, the answer is equal to the square of the first term minus the square of the second term.

$$(2m + 3n)(2m - 3n) = 4m^2 - 9n^2$$

The same is done to the following examples.

Example 6

Expand and simplify: $(3a + \frac{1}{5}b)(3a - \frac{1}{5}b)$

$$\text{Solution: } (3a + \frac{1}{5}b)(3a - \frac{1}{5}b) = 9a^2 - \frac{1}{25}b^2$$

Example 7

Expand and simplify: $(4x + 6y)(4x - 6y)$

$$\text{Solution: } (4x + 6y)(4x - 6y) = 16x^2 - 36y^2$$

NOW DO PRACTICE EXERCISE 6

**Practice Exercise 6**

Expand and then simplify the following:

(a) $(4m + n)(m + n)$

Answer: _____

(b) $(a + 2b)(2a + b)$

Answer: _____

(c) $(r + s)(2r - 3s)$

Answer: _____

(d) $(3x - y)(12x - y)$

Answer: _____

(e) $(5a - \frac{1}{4}b)(4a + b)$

Answer: _____

(f) $(5a - 4b)(6a - 7b)$

Answer: _____

(g) $(c + d)(c - d)$

Answer: _____

(h) $(5x + 2y)(5x - 2y)$

Answer: _____

(i) $(2w + z)^2$ Hint: $(2w + z)^2 = (2w + z)(2w + z)$

Answer: _____

(j) $(4t - 2s)^2$

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.
--

TOPIC 1: SUMMARY



This summarizes some of the important ideas and concepts to remember.

- **Numerical expression** is an expression that contains only constants separated by four fundamental operations (addition, subtraction, multiplication and division).
- **Algebraic expression** is an expression which is a combination of algebraic terms or group of numerical and algebraic terms which are added or subtracted from one another.
- **Collecting like terms** is a process used to simplify algebraic expressions using addition and subtraction of the coefficient of terms.
- **Like terms** are two or more terms that have exactly the same variable with the same exponent.
- **Expanding algebraic expressions** means re-writing the expressions in an equivalent form by removing grouping symbols.
- **Grouping symbols** are symbols used to show which part of the expression should be done first. There are three types of grouping symbols: parentheses (), brackets [], and braces { }.
- **A negative algebraic term** is a term that has a negative (minus) sign in front of either a single variable or a combination of a number and one or more variables.
- When **expanding and simplifying algebraic expressions**:
 - If the grouping symbol is preceded by a plus (+) sign, remove the grouping symbol and collect like terms where necessary.
 - If the grouping symbol is preceded by a minus (-) sign, remove the grouping symbol and change the signs of the terms inside the grouping symbol by their opposite (+ to - or - to +) then collect like terms where necessary.
 - If the expression involves a series of grouping symbols, start by removing from the innermost grouping symbol and apply the first two rules.
- **Binomial expression** is an expression consisting of two terms.
- The product of two binomials is called a **binomial product**.
- **Direct Expansion Method** is used in expanding binomial product. This means that "to expand a binomial product, multiply the terms of the second binomial by each term of the first binomial and then simplify by collecting like terms.
- The **product of the sum and difference of two terms** is equal to the square of the first term minus the square of the second term.
- To **expand and simplify two binomials with two variables**, multiply the two terms of the second binomial by the two terms in the first binomial. Then collect like terms where necessary.

REVISE LESSON 1 TO 6. THEN DO TOPIC 1 TEST 1 IN ASSIGNMENT BOOK 3.

ANSWERS TO PRACTICE EXERCISES 1 – 6**Practice Exercise 1**

1. (a) $6pqy$ (e) $40j^2k$
 (b) $5st$ (f) $7b^2m^2$
 (c) $3a^2b$ (g) $30f^2gh^2$
 (d) $60x^2y^2$ (h) 0
2.

<u>b</u> 1) $8x - 3x$ <u>e</u> 2) $3c + 9y - 5c$ <u>g</u> 3) $-4m + 5m - 7mn$ <u>d</u> 4) $6ab + 4bc - 3ab + bc$ <u>a</u> 5) $7x^2y - 2x^2y + 5xy^2 - 3xy^2$ <u>f</u> 6) $-4m - 7mn + 8n$ <u>c</u> 7) $8c + 9y - 5c$	a. $5x^2y + 2xy^2$ b. $5x$ c. $3c + 9y$ d. $3ab + 5bc$ e. $-2c + 9y$ f. $-4m - 7mn + 8n$ g. $m - 7mn$
--	---
3. (a) $-b + 3$
 (b) $n + 3t$
 (c) $-4abc + 7$
 (d) $-a^2b + ab^2 + 8$
 (e) $2a + 2ab + b$
 (f) $2a - 2b - 5c + 8$
 (g) $3p^2 - 2p - 12$
 (h) $2x^2 + 9x + 9$

Practice Exercise 2

1. $15a + 20$
 2. $9x + 108$
 3. $4m + 17$
 4. $8c + 8d + 32$
 5. $1 + 7x + 35y$ or $7x + 35y + 1$
 6. $13x + 8y + 4$
 7. $6a + 42b + 24c + 12$
 8. $6x^2 + 90x$
 9. $3.6x + 1.8y + 3.6$

10. $15y^2 + 11$
 11. $24x + 4y + 18$
 12. $x^2 + 4x^2y + 4xy^2 + 29xy$
 13. $3x^2 + 10x + 28$
 14. $m^2 + 15m + mn + 28n + mp + 36p$
-

Practice Exercise 3

- | | |
|--------------|------------------|
| 1. 36 | 6. $-42b^2c$ |
| 2. -42 | 7. $wxyz$ |
| 3. $9m$ | 8. $90a^2bc$ |
| 4. $-8q^2$ | 9. $324mn^2p$ |
| 5. $-32r^2s$ | 10. $-12a^2bcde$ |
-

Practice Exercise 4

- | | |
|--------------------|---------------------------|
| 1. (a) $6x + 18$ | (f) $5m - 20$ |
| (b) $-3m + 3n$ | (g) $x^2 + xy$ |
| (c) $-x^2 - 2x$ | (h) $y^3 - 2y$ |
| (d) $6x - 8$ | (i) $3t^2 + 4t$ |
| (e) $-3x - 3y$ | (j) $4y - 20$ |
| 2. (a) $10p - 2q$ | (f) $6v + 27w$ |
| (b) $15d + 21e$ | (g) $21q + 32r$ |
| (c) $-8e - 4f$ | (h) $m^2 - 2m + 2$ |
| (d) $3x - 6y$ | (i) $-x + 18$ or $18 - x$ |
| (e) $21t - 3u$ | (j) $5m - 13$ |
| 3. (a) $-7y - 7$ | |
| (b) $5x - 2y + 10$ | |
-

Practice Exercise 5

- (a) $x^2 + 11x + 30$
- (b) $3x^2 + 12x + 12$
- (c) $21x^2 + 44x + 15$
- (d) $y^2 + 5y - 6$
- (e) $12a^2 - 14a - 10$
- (f) $w^2 - 5w + 6$

- (g) $3x^2 + 20x + 12$
(h) $25x^2 + 30x + 9$
(i) $64b^2 - 80b + 25$
(j) $-12y^2 - 16y + 35$
-

Practice Exercise 6

- | | |
|------------------------------------|----------------------------|
| (a) $4m^2 + 5mn + n^2$ | (f) $30a^2 - 59ab + 28b^2$ |
| (b) $2a^2 + 5ab + 2b^2$ | (g) $c^2 - d^2$ |
| (c) $2r^2 - rs - 3s^2$ | (h) $25x^2 - 4y^2$ |
| (d) $36x^2 - 15xy + y^2$ | (i) $4w^2 + 4wz + z^2$ |
| (e) $20a^2 + 4ab - \frac{1}{4}b^2$ | (j) $16t^2 - 16st + 4s^2$ |
-

END OF TOPIC 1

TOPIC 2

FACTORIZING ALGEBRAIC EXPRESSIONS

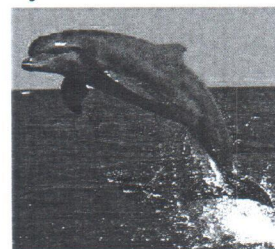
- | | |
|-------------------|--|
| Lesson 7: | Factorizing Simple Algebraic Expressions |
| Lesson 8: | Factorizing Quadratic: Common Monomial Factor |
| Lesson 9: | Factorizing Quadratic: Trinomials |
| Lesson 10: | Further Quadratic Factorization |
| Lesson 11: | Simplifying Quotients by Factorization |

TOPIC 2: FACTORIZING ALGEBRAIC EXPRESSIONS

Introduction



Any trip to Sea World is not complete until you have seen a dolphin leap high above the water to grab a fish. Have you ever wondered how long a dolphin can stay in the air after jumping out of the water? In a standard park pool, dolphins can reach speeds of 24 feet per second. If a dolphin leaves the water traveling at 24 feet per second and it jumps h feet above the water after t seconds, then its formula is



$$h = 24t - 16t^2$$

When the dolphin returns to the water, it is 0 feet above the water. Thus, by substituting, the formula becomes $0 = 24t - 16t^2$. An obvious solution of the equation is when $t=0$, but when time is 0, the dolphin has not jumped out of the water!

In this Topic, we will learn a technique called **factoring** that will give us the solution.

First, you will revise the meaning of prime numbers, factors, prime factorization and highest common factor which you already studied in Grade 8 and 9 Mathematics.

Then we will look at the meaning of factorization and the basic methods of factorizing algebraic expressions.

You will also learn the meaning of quadratic expression and how to factorize them.

And finally, you will learn to apply factorization in simplifying and writing complicated algebraic expressions into a much simpler form.

Lesson 7: Factorizing Simple Algebraic Expressions



In the previous topic, you learnt how to expand algebraic expressions.



In this lesson, you will:

- revise the terms factors and greatest common factor.
- define factorization
- factorize simple algebraic expressions.

When two numbers are multiplied together, the answer is called a product. The numbers that were multiplied together are called factors of the product.

“Factors” are the numbers you multiply together to get another number.

A good way to find all the factors of a number is by finding all of that number’s factor pairs. A **factor pair** of a number is any pair of two numbers that, when multiplied together, equal that number.

For example, 24 have four pairs of factors:

$$\begin{aligned} 1 \cdot 24 &= 24 \\ 2 \cdot 12 &= 24 \\ 3 \cdot 8 &= 24 \\ 4 \cdot 6 &= 24 \end{aligned}$$

(1, 2, 3, 4, 6, 8, 12, and 24 are factors of 24)

Notice that if we divide the product 24 by any of its factors 1, 2, 3, 4, 6, 8, 12, and 24 the quotient is exact with no remainder.

Example: $\frac{24}{1} = 24$; $\frac{24}{2} = 12$; $\frac{24}{3} = 8$; $\frac{24}{4} = 6$; $\frac{24}{6} = 4$; $\frac{24}{8} = 3$; $\frac{24}{12} = 2$; $\frac{24}{24} = 1$

Thus, we also say

The **factors of a number** are all those numbers that can divide evenly into the number with no remainder.

The greatest factor of a number is the number itself, so you can always list all the factors of any number because you have a stopping point.

Follow these steps to list all the factors of a number:

1. Begin the list with 1, leave some space for other numbers, and end the list with the number itself.
2. Test whether 2 is a factor — that is, see whether the number is divisible by 2. If it is, add 2 to the list, along with the original number divided by 2 as the second-to-last number on the list.

3. Test the number 3 in the same way.
4. Continue testing numbers until the beginning of the list meets the end of the list.

Examples

1. The factors of 12 are 1, 2, 3, 4, 6, and 12.
2. The factors of 42 are 1, 2, 3, 6, 7, 14, 21, and 42
3. The factors of 27 are 1, 3, 9 and 27.

Note: (i) 1 is always a factor.
(ii) A number is a factor of itself.

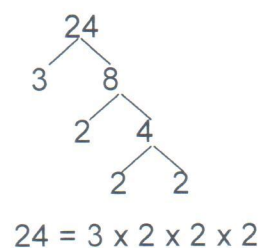
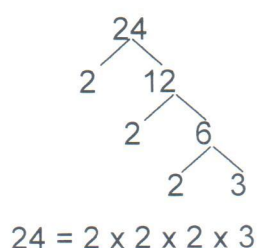
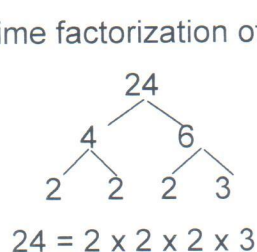
When a number has no other pairs of factors except itself and 1, it is called a **prime number**.

Likewise, when a factor has no other pair of factor except itself and 1, it is called a **prime factor**.

Prime factorization is finding which prime numbers multiply together to make the original number.

To find the prime factorization of any number, we use the factor tree.

The prime factorization of 24:



Greatest Common Factor (GCF)

The **Greatest Common Factor (GCF)** also known as **highest common factor (HCF)** of two or more numbers is the highest number that divides exactly into two or more numbers.

Below are two ways of how to find the GCF of a pair of numbers.

1. By listing all the factors and getting the largest factor common to both the given numbers.

For example: What is the GCF of 24 and 36.

The factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

The factors of 36 = 1, 2, 3, 4, 6, 9, 12, 18, 36

Looking through the above two list of numbers, we see that the factors common to both 24 and 36 are: 1, 2, 3, 4, 6, and 12. The highest is 12.

Therefore, GCF of 24 and 36 is 12.

2. By prime factorization

For example: The prime factors of $24 = 2 \times \boxed{2 \times 2 \times 3}$

The prime factors of $36 = \boxed{2 \times 2 \times 3} \times 3$

Common prime factors $= 2 \times 2 \times 3$

Therefore, the GCF $= 2 \times 2 \times 3 = 12$

An algebraic expression can also be factored into one or more factors.

When factorizing algebraic expressions, we rewrite the expression as a product of its greatest or highest common monomial factor and the remaining terms. These remaining terms are enclosed within parentheses or brackets.

The **Greatest Common Factor** (GCF) of an algebraic expression is the largest monomial term which will divide evenly into all the terms in the expression.

For example:

1. The **GCF** for $6x$ and $9x$ is **$3x$** because $3x$ is the largest monomial term which will divide evenly into both $6x$ and $9x$.
2. The GCF for $36x^3$, $42x^4$ and $24x^5$ is **$6x^3$**
3. The GCF for $15x^2y^3z$, $45x^6y^4c$, and $30x^6y^5d$ is **$15x^2y^3$**

Now, let us look at what would the following algebraic expressions look like.

Example 1

Factorize $ax + bx$.

Solution: In $ax + bx$, x is common and also a common factor.

Hence, $ax + bx = x(a + b)$

Notice that the solution is written as the product of the GCF and the original expression after you 'factored out' the GCF.

Example 2

Factorize: $27x^4y^3 - 54x^3y^5$

Solution: In $27x^4y^3 - 54x^3y^5$, the GCF is **$27x^3y^3$**

27 is the GCF of 27 and 54 , x^3 and y^3 are common in both of the terms.

Divide both terms of the expression by their GCF ($27x^3y^3$)

So, the solution is written: $27x^4y^3 - 54x^3y^5 = 27x^3y^3(x - 2y^2)$

Now let us do other examples.

Example 3

Factorize $7p^2 - 14pq + 7pr$

Solution: The GCF is $7p$.

7 is the GCF of 7 and 14, p is common in all of the terms. q and r are not common because they are not in all the terms.

Divide all terms by the GCF ($7p$)

So the solution is written as: $7p(p - 2q + r)$

Example 4

Factorize $6a^3bc + 18a^2b^4d - 12a^6b^2e$

Solution: The GCF is $6a^2b$

6 is the GCF of 6, 18 and 12. c , d , e are not common because they are not in all the terms.

Divide all terms by the GCF ($6a^2b$)

So, the solution is written as: $6a^2b(ac + 3b^3d - 2a^4be)$

Example 5

Factorize $9a^2b^2 + 11ab^3 + 2a^4b$

Solution: The GCF is ab

ab is common in all the terms. 9, 11 and 2 have no common factor except 1.

Divide all the terms by the GCF (ab)

So, the solution is written as: $ab(9ab + 11b^2 + 2a^3)$

NOW DO PRACTICE EXERCISE 7

**Practice Exercise 7**

1. Write down all of the factors (apart from 1) of:

(a) 16

(b) 40

(c) 17

Answer: _____

Answer: _____

Answer: _____

2. Write down all of the **basic** factors of the following expressions:

(a) $5x$

(c) $4xy$

Answer: _____

Answer: _____

(b) $12a$

(d) $14abc$

Answer: _____

Answer: _____

3. Write down the highest common factors of:

(a) 6 and 8

(e) $6r - 9t$

Answer: _____

Answer: _____

(b) 8 and 16

(f) $m + mn$

Answer: _____

Answer: _____

(c) 9, 18 and 27

(g) $abc - 2ab + a$

Answer: _____

Answer: _____

(d) $4x + 4y$

(h) $60x + 20xy + 30xyz$

Answer: _____

Answer: _____

4. Factorize using the GCF.

(a) $4a + 12b - 16c$

(b) $18x - 12y - 30z$

(c) $13x^2 - 5x^3 + 8x^7$

(d) $9a^2b^2 + 11ab^3 + 2a^4b$

(e) $4x^3 + 6x^4 + 12x^2$

(f) $21ab^3 + 24zb^2$

(g) $14x^5 - 63x^4 + 35x^3 + 21x^2$

(h) $16a^2b^3c + 32a^3b^2d - 40a^2b^4e$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 8: Factorizing Quadratics: Common Monomial Factor



You have revised the terms 'factor, greatest common factor, prime numbers and factorization' in the previous lesson. You also learnt to factorize simple algebraic expressions.



In this lesson, you will:

- discuss and identify quadratic expressions
- factorize quadratic expressions with common variable

Let us define a quadratic expression in general.

An algebraic expression in which the highest exponent is 2 is referred to as a **Quadratic Expression**.

The above definition implies that in a Quadratic Expression, the exponent on the variable can neither be greater than 2 nor smaller than 2.

For example, the following algebraic expressions are quadratic expressions because their highest exponent is exactly equal to 2:

1. $x^2 + 3x + 5$
2. $1 + x^2$
3. $y^2 - 5xy$

But the following algebraic expressions are **not** quadratic expressions because they contain exponents which are higher than two.

1. $x^3 + x^2 + 5$
2. $2b^4 + 4b^2 + 8$

Note that a quadratic expression is also called as a **second degree expression** because the degree, or highest exponent, of a quadratic expression is always two.

Factorizing quadratic expressions with common variable

When factorizing quadratic expressions with common variable, you use the same method as in Lesson 7. Meaning, you have to rewrite the expression as a product of the GCF and the remaining terms of the expression.

For example: Factorize the quadratic expression $x^2 + 2x$.

Solution: The GCF is x , because it is common in both terms.
Factoring out x from the expression, the remaining term will be $x + 2$.
Rewriting them as product we have: $x(x + 2)$
So, $x^2 + 2x = x(x + 2)$

Here are other examples.

Example 2

Factorize: $16m^2 + 12m$

Solution: The largest monomial factor is $4m$.
 4 is the GCF of 16 and 12 , m is the common variable in both terms.
Divide both terms of the quadratic expressions by the GCF ($4m$).

Hence, the solution is written as: $16m^2 + 12m = 4m(4m + 3)$

Example 3

Factorize: $40u^2 - 16u$

Solution: The largest monomial factor is $8u$.
 8 is the GCF of 40 and 16 , u is the common variable in both terms.
Divide both terms of the quadratic expressions by the GCF ($8u$).

Hence, the solution is written as: $40u^2 - 16u = 8u(5u + 2)$

Example 4

Factorize: $16a^2b - 8ab - 24a$

Solution: The largest monomial factor is $8a$.
 8 is the GCF of 16 , 8 and 24 , a is the common variable in all terms
Divide both terms of the quadratic expressions by the GCF ($8a$).

Hence, the solution is written as: $16a^2b - 8ab - 24a = 8a(ab - b - 3)$

Example 5

Factorize: $-9e^2 - 18efg + 9e$

Solution: The largest monomial factor is $9e$.
 9 is the GCF of 9 and 18 , e is the common variable in all terms
Divide both terms of the quadratic expressions by the GCF ($9e$).

Hence, the solution is written as: $-9e^2 - 18efg + 9e = 9e(-e - 2fg + 1)$

NOW DO PRACTICE EXERCISE 8

**Practice Exercise 8**

1. Factorize the following quadratic expressions:

(a) $c^2 + c$

Answer: _____

(b) $12x^2 - x$

Answer: _____

(c) $5a^2 - 5a$

Answer: _____

(d) $5p^2 + 10p$

Answer: _____

(e) $8x^2 - 12x$

Answer: _____

(f) $m^2 + mb$

Answer: _____

(g) $s^2 - 2st + 3s$

Answer: _____

(h) $6a^2 + 12ab + 6ac$

Answer: _____

(i) $14p^2 - 28pq - 35pr$

Answer: _____

(j) $-7f^2 - 14fgh + 7f$

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 9: Factorizing Quadratics: Trinomials



You have learned in the previous lesson how to factorize simple quadratic expressions whose terms contain common monomial factor.



In this lesson, you will:

- factorize quadratic expressions containing three terms that do not have common monomial factors.

So far, you have learned to factorize quadratic expressions which contain a common monomial factor in Lesson 8.

For example: $5x^2 + 3x$ has a common monomial factor x .

On factoring, we have rewritten the quadratic expression $5x^2 + 3x = x(5x + 3)$

You have also learned how to factorize quadratic expressions containing three terms with common monomial factor.

For example: $6p^2 - 12pq + 18p$ has three terms containing monomial factor $6p$.

On factoring we have: $6p^2 - 12pq + 18p = 6p(p - 2q + 3)$

Notice that in the above examples, the factorized form contains only one parenthesis.

We will now factorize a different type of quadratics: (those which contain three terms but where we have one term containing a variable another term containing the same variable squared and a third term containing only a number).

We call this type of quadratic expression as **quadratic trinomial** and they are of the form $x^2 \pm bx \pm c$.

For example:

$$x^2 + 5x + 6$$

$$c^2 + 8c + 15$$

$$y^2 - 7y + 10$$

When factorizing quadratic expression of this type, we need two pairs of parentheses or brackets. It is not as easy as what you did in the previous lesson.

Nearly all the expansion of binomials you did in Topic 1 had answers of the examples shown above.

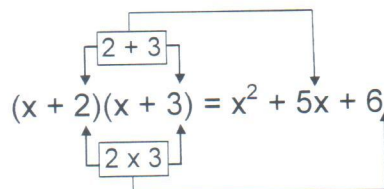
For example if we expand $(x + 2)(x + 3)$ we have:

$$\begin{aligned}(x+2)(x+3) &= x(x+3) + 2(x+3) \\ &= x^2 + 3x + 2x + 6 \\ &= x^2 + 5x + 6\end{aligned}$$

Since the product of $(x + 2)$ and $(x + 3) = x^2 + 5x + 6$ then, $(x + 2)$ and $(x + 3)$ are factors of $x^2 + 5x + 6$.

So, the factorized form of the trinomial $x^2 + 5x + 6$ is $(x + 2)(x + 3)$.

Notice that the two numerical factors 2 and 3, which give a product of 6 must also add together to give a sum of +5 since the coefficient (the number in front) of the middle term, $5x$, as shown in the diagram below.



This in fact tells us that expanding and factoring are opposites.

Expanding is easy, but factoring can often be tricky.

For example, in order to factorize the quadratic expression $x^2 + 3x - 10$, we will need two pairs of brackets or parentheses as shown.

$$x^2 + 3x - 10 = (\quad)(\quad)$$

Now we need to fill in the statement above.

The first term of the quadratic expression is x^2 . This can be obtained by putting x into each pair of parentheses because $x \cdot x$ is x^2 . So the factors of x^2 are x and x .

Hence, we have, $x^2 + 4x - 21 = (x \quad)(x \quad)$

Now let us consider the last term of the quadratic expression, -21 , in the same way.

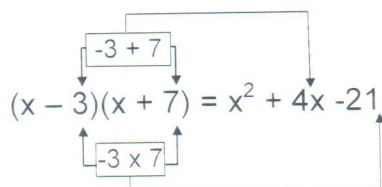
We need to find two numbers that multiply together to give -21 and add together to give 4 . In this case, we need to list all the factors of -21 .

Here they are:

-1 and 21	1 and -21
-3 and 7	3 and -7

We have a choice of pairs of factors to put in the parentheses. In this case the only pair of numbers which have a product of -10 and a sum of 4 are -3 and 7 .

So we have



So numbers we put in the parentheses are -3 and 7 .

Therefore, the factorized form of $x^2 + 4x - 21 = (x - 3)(x + 7)$.

Here are other examples.

Example 2

Factorize $y^2 - 4y - 32$.

Solution: We need to fill in the parentheses in the statement:

$$y^2 - 4y - 32 = (\quad)(\quad)$$

The first term of the quadratic is y^2 . We can obtain by putting y into each pair of the parentheses since $y \cdot y = y^2$

So, $y^2 - 4y - 32 = (y \quad)(y \quad)$

Now consider the last term in the quadratic, -32 .

The factors of -32 are:	-1 and 32	1 and -32
	-4 and 8	4 and -8
	-2 and 16	2 and -16

The only pair of numbers which have a product of -32 and a sum of -4 are 4 and -8 .

So the numbers we put in the parentheses are 4 and -8 .

Therefore, in factorized form: $y^2 - 4y - 32 = (y + 4)(y - 8)$

Example 3

Factorize $x^2 + 7x + 12$

Solution: We need to fill in the parentheses in the statement:

$$x^2 + 7x + 12 = (\quad)(\quad)$$

The first term of the quadratic is x^2 . We can obtain by putting x into each pair of the parentheses since $x \cdot x = x^2$

So, $x^2 + 7x + 12 = (x \quad)(x \quad)$

Now consider the last term in the quadratic, $+12$.

The factors of $+12$ are:	1 and 12	-1 and -2
	2 and 6	-2 and -6
	3 and 4	-3 and -4

The only pair of numbers which have a product of $+12$ and a sum of $+7$ are 3 and 4 .

So the numbers we put in the parentheses are 3 and 4 .

Therefore, in factorized form: $x^2 + 7x + 12 = (x + 3)(x + 4)$

Here is another example.

Factorize $m^2 + 13m + 36$

Solution: We need to fill in the parentheses in the statement:

$$m^2 + 13m + 36 = (\quad)(\quad)$$

The first term of the quadratic is m^2 . We can obtain by putting m into each pair of the parentheses since $m \cdot m = m^2$

So, $m^2 + 16m - 36 = (m \quad)(m \quad)$

Now consider the last term in the quadratic, $+12$.

The factors of $+12$ are:	-1 and 36	1 and -36
	-2 and 18	2 and -18
	-3 and 12	3 and -12
	-4 and 9	4 and -9

The only pair of numbers which have a product of -36 and a sum of $+16$ are -2 and 18 .

So the numbers we put in the parentheses are -2 and 18 .

Therefore, in factorized form: $m^2 + 16m - 36 = (x - 2)(x + 18)$

To check if an answer is correct, expand the brackets or parentheses and see if the result is the same as the original expression. If this is not so then a mistake has been made.

For example, in our last example the answer is $(m - 2)(m + 18)$ and the original expression is $m^2 + 16m - 36$.

To check if the answer is correct proceed as follows;

Original expression: $m^2 + 16m - 36$ and answer is: $(m - 2)(m + 18)$

Expanding the answer:

$$(m - 2)(m + 18) = m(m+18) - 2(m + 18)$$

$$= m^2 + 18m - 2m - 36$$

$$= m^2 + 16m - 36 \leftarrow \text{Same as the original expression, so it is correct.}$$

NOW DO PRACTICE EXERCISE 9



Practice Exercise 9

1. Find the total for each of the following table machine. The first one is done for you.

(a)

x	n	+4
n	n^2	$4n$
-2	$-2n$	-8

The total for this table is $n^2 + 2n - 8$

(b)

x	a	-7
a	—	—
-9	—	—

The total for this table is _____.

(c)

x	m	-7
m	—	—
2	—	—

The total for this table is _____.

2. Copy and complete the following statements:

(a) $x^2 + 7x + 12 = (x + 4)(\quad)$

(b) $x^2 + __x + 10 = (x + 5)(x + 2)$

(c) $x^2 + 6x + __ = (x + 4)(x + 2)$

(d) $a^2 + __a + 5 = (a + 1)(\quad)$

(e) $c^2 - 10c - __ = (c + 1)(c - 11)$

3. Factorize these expressions below.

(a) $n^2 + 8n + 12$

Answer: _____

(b) $x^2 + 6x + 9$

Answer: _____

(c) $c^2 - 9c + 8$

Answer: _____

(d) $y^2 - 7x - 18$

Answer: _____

(e) $r^2 + 12r + 11$

Answer: _____

(f) $x^2 + 9x + 14$

Answer: _____

(g) $t^2 - 5t + 6$

Answer: _____

(h) $x^2 - x - 20$

Answer: _____

(i) $w^2 - 8w + 15$

Answer: _____

(j) $z^2 + 10z - 24$

Answer: _____

4. Given the following expressions.

$(k + 2)$

$(k - 7)$

$(k - 2)$

$(k + 14)$

$(k + 7)$

$(k - 1)$

Which pair of expressions multiplies to give $k^2 + 5k - 14$?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.
--

Lesson 10: Further Quadratics Factorization



You have learned to factorize quadratic expressions of the form $x^2 \pm bx \pm c$ in the previous lesson.



In this lesson, you will:

- factorize difference of two squares.

We will extend the method of factorization you learnt in the previous lesson for quadratic expressions which only contain a squared variable term and a negative number.

For example, the expression $x^2 - 9$ contains only a squared variable term, (x^2) and a negative number, (-9).

To factorize this expression $x^2 - 9$, for example, we need to fill in the parentheses in the statement

$$x^2 - 9 = (\quad)(\quad)$$

To be able to use the method used in the previous lesson for quadratic trinomials, we will rewrite $x^2 - 9$ as $x^2 + 0x - 9$. Adding in a zero ($0x = 0$) does not change the value of the original expression.

Since now $x^2 - 9 = x^2 + 0x - 9$ the coefficient of the middle term is, $0x$ is 0.

Since $x \cdot x = x^2$, then the first term in the quadratic x^2 can be obtained by putting x in each of the parentheses.

So,
$$x^2 - 9 = (x \quad)(x \quad)$$

Now consider the last term in the expression, -9. We need to find two numbers that multiply to give a product of -9 and also add to give a sum of 0, since the middle term, $0x$ is 0.

The factors of -9 are: -1 and 9 1 and -9 -3 and 3

The only pair of factors which have a product of -9 and a sum of 0 are -3 and 3. So, these are the numbers we put into each parentheses.

So, the factorized form:
$$x^2 + 0x - 9 = (x - 3)(x + 3)$$

Therefore,
$$x^2 - 9 = (x - 3)(x + 3)$$

As in the previous lesson,

To check if an answer is correct, expand the brackets or parentheses and see if the result is the same as the original expression. If this is not so then a mistake has been made.

Check: $(x - 3)(x + 3) = x(x + 3) - 3(x + 3)$
 $= x^2 + 3x - 3x - 9$
 $= x^2 - 9$

Notice that in the above example, the two parentheses contain the same number but have different signs. This must be so, since the sum of the two numbers must be zero.

Notice also that the expression $x^2 - 9$ is a difference of two squares. Since x^2 is the square of x and 9 is the square of 3.

Hence, in this lesson, we factorize a difference of two square terms.

Hence when factorizing a difference of two square terms, the second bracket is the same as the first bracket except minus sign.

The terms of the expression cancel each other out when expanded.

Now we can generalize the rule when factoring a quadratic expression which is a difference of two square terms.

The factors of the difference of two square terms are the sum and difference of their square roots.

Now let us have some examples.

Example 1

Factorize $x^2 - 25$

Solution: The square root of x^2 is x , since $x \cdot x = x^2$
The square root of 25 is 5, since $5 \cdot 5 = 25$

Writing the sum $(x + 5)$ and the difference $(x - 5)$.

Therefore, the factorized form: $x^2 - 25 = (x + 5)(x - 5)$

Check: $(x + 5)(x - 5) = x(x + 5) - 5(x + 5)$
 $= x^2 + 5x - 5x - 25$
 $= x^2 - 25$

Example 2

Factorize $a^2 - 144$

Solution: The square root of a^2 is a , since $a \cdot a = a^2$
The square root of 144 is 12, since $12 \cdot 12 = 144$

Writing the sum $(a + 12)$ and the difference $(a - 12)$.

Therefore, the factorized form: $a^2 - 144 = (a + 12)(a - 12)$

Check: $(a + 12)(a - 12) = a(a + 12) - 12(a + 12)$
 $= a^2 + 12a - 12a - 144$
 $= a^2 - 144$

Example 3

Factorize $b^2 - 256$

Solution: The square root of b^2 is b , since $b \cdot b = b^2$
The square root of 256 is 16, since $16 \cdot 16 = 256$

Writing the sum $(b + 16)$ and the difference $(b - 16)$.

Therefore, the factorized form: $b^2 - 256 = (b + 16)(b - 16)$

Check: $(b + 16)(b - 16) = b(b + 16) - 16(b + 16)$
 $= b^2 + 16b - 16b - 256$
 $= b^2 - 256$

Example 4

Factorize $4x^2 - 9y^2$

Solution: The square root of $4x^2$ is $2x$, since $2x \cdot 2x = 4x^2$
The square root of $9y^2$ is $3y$, since $3y \cdot 3y = 9y^2$

Writing the sum $(2x + 3y)$ and the difference $(2x - 3y)$.

Therefore, the factorized form: $4x^2 - 9y^2 = (2x + 3y)(2x - 3y)$

Check: $(2x + 3y)(2x - 3y) = 2x(2x + 3y) - 3y(2x + 3y)$
 $= 4x^2 + 6xy - 6xy - 9y^2$
 $= 4x^2 - 9y^2$

Example 5

Factorize $9x^2 - 16y^2$

Solution: The square root of $9x^2$ is $3x$, since $3x \cdot 3x = 9x^2$
The square root of $16y^2$ is $4y$, since $4y \cdot 4y = 16y^2$

Writing the sum $(3x + 4y)$ and the difference $(3x - 4y)$.

Therefore, the factorized form: $9x^2 - 16y^2 = (3x + 4y)(3x - 4y)$

Check: $(3x + 4y)(3x - 4y) = 3x(3x + 4y) - 4y(3x + 4y)$
 $= 9x^2 + 12xy - 12xy - 16y^2$
 $= 9x^2 - 16y^2$

Example 6

Factorize $4m^2 - 36n^2$

Solution: There is a common factor of 4 that can be factored out first in this problem, to make the problem easier.

$$\text{Hence, we have } 4m^2 - 36n^2 = 4(m^2 - 9n^2)$$

In the factor $(m^2 - 9n^2)$, is a difference of two squares

The square root of $m^2 = m$, since $m \cdot m = m^2$

The square root of $9n^2 = 3n$, since $3n \cdot 3n = 9n^2$

$$\text{Factorized form of } m^2 - 9n^2 = (m + 3n)(m - 3n)$$

Therefore, the factorized form of $4m^2 - 36n^2 = 4(m + 3n)(m - 3n)$

NOW DO PRACTICE EXERCISE 10

**Practice Exercise 10**

1. (a) Write down two numbers whose product is 64 and whose sum is zero.

Answer: _____

Hence, factorize $n^2 - 64$

Answer: _____

- (b) Write down two numbers whose product is 121 and whose sum is zero.

Answer: _____

Hence, factorize $w^2 - 121$

Answer: _____

2. Factorize the following.

(a) $x^2 - 36$

Answer: _____

(b) $y^2 - 49$

Answer: _____

(c) $p^2 - 64$

Answer: _____

(d) $81 - c^2$

Answer: _____

(e) $4x^2 - 25$

Answer: _____

3. Factorize the following completely.

(a) $5b^2 - 5$

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.
--

Lesson 11: Simplifying Quotients by Factorization



You have learned to factorize quadratic expression which is a difference of two squares in the previous lesson.



In this lesson, you will:

- factorize algebraic fractions
- simplify quotients by cancelling after factorizing algebraic expressions.

Quotients in Algebra are often called **algebraic fractions** or **rational expressions**.

The numerators and the denominators of an algebraic fraction are called **terms**. Since we may multiply both terms, then, symmetrically we may divide both terms.

For example $\frac{ax}{ay} = \frac{x}{y}$

“Both the numerator and denominator may be divided by a common factor.”

When we do that, we say that we have simplified the fraction to its simplest form.

Example 2

Simplify: $\frac{5x}{5y}$

Solution: 5 is a common factor of the numerator and the denominator. Therefore we can divide each of them by 5.

$$\text{Hence, } \frac{5x}{5y} = \frac{x}{y}$$

One often hears that we have “cancelled” the 5’s, but that can be an incorrect expression, as the following examples will show.

Example 3

Simplify: $\frac{5+x}{5+y}$

Solution: This cannot be simplified. We cannot “cancel” the 5’s, because 5 is not a factor of either the numerator or the denominator. In both of them 5 is a term. We cannot cancel terms.

$\frac{5+x}{5+y}$ is in its simplest form since the numerator and the denominator have no common factors.

Example 4

Simplify: $\frac{3w + 6x + 9y}{12z}$

Solution: In this example, every term in the numerator and denominator has a common factor 3.

$$\begin{aligned}\text{So, } \frac{3w + 6x + 9y}{12z} &= \frac{3(w + 2x + 3y)}{3(4z)} \\ &= \frac{w + 2x + 3y}{4z}\end{aligned}$$

Example 5

Simplify: $\frac{8x}{8x + 10}$

Solution: In this example, every term in the numerator and denominator has a common factor 2.

$$\begin{aligned}\text{So, } \frac{8x}{8x + 10} &= \frac{2(4x)}{2(4x + 5)} \quad \leftarrow 2 \text{ cancelled} \\ &= \frac{4x}{4x + 5}\end{aligned}$$

There is no more dividing, we cannot cancel 4x's, because 4x is not a factor of the denominator. 4x is not a factor of 5.

Therefore, the simplified form: $\frac{8x}{8x + 10} = \frac{4x}{4x + 5}$

Example 6

Simplify: $\frac{x - 3}{6(x - 3)}$

Solution: In this problem, we can view $x - 3$ as a factor of the numerator because $x - 3 = 1(x - 3)$.

$$\begin{aligned}\text{Hence, } \frac{x - 3}{6(x - 3)} &= \frac{1(\cancel{x - 3})}{6(\cancel{x - 3})} \quad \leftarrow (x - 3) \text{ cancelled} \\ &= \frac{1}{6}\end{aligned}$$

Therefore, the simplified form: $\frac{x - 3}{6(x - 3)} = \frac{1}{6}$

Example 7

Simplify: $\frac{x^2 - x - 6}{x^2 - 4x + 3}$

Solution: In this expression, there are no common factors but we can make factors.

$$\text{The factors of } x^2 - x - 6 = (x + 2)(x - 3)$$

$$\text{The factors of } x^2 - 4x + 3 = (x - 1)(x - 3)$$

$$\begin{aligned}\text{Hence, } \frac{x^2 - x - 6}{x^2 - 4x + 3} &= \frac{(x + 2)(x - 3)}{(x - 1)(x - 3)} \quad \leftarrow (x - 3) \text{ cancelled} \\ &= \frac{(x + 2)}{(x - 1)}\end{aligned}$$

$$\text{Therefore, the simplified form: } \frac{x^2 - x - 6}{x^2 - 4x + 3} = \frac{(x + 2)}{(x - 1)}$$

Example 8

Simplify: $\frac{2a + ab}{2 + b}$

Solution: $2a + ab = a(2 + b)$

$$\begin{aligned}\text{Hence, } \frac{2a + ab}{2 + b} &= \frac{a(2 + b)}{(2 + b)} \quad \leftarrow (2 + b) \text{ cancelled} \\ &= \frac{a(1)}{1} \\ &= a\end{aligned}$$

$$\text{Therefore, } \frac{2a + ab}{2 + b} = a$$

NOW DO PRACTICE EXERCISE 11

**Practice Exercise 11**

1. Simplify the following:

(b) $\frac{4x + 4}{4}$

Answer: _____

(c) $\frac{5x}{10x + 15}$

Answer: _____

(d) $\frac{12x - 18y + 21z}{6y}$

Answer: _____

(e) $\frac{5m + 10}{m + 2}$

Answer: _____

(f) $\frac{2x - 2y}{3x - 3y}$

Answer: _____

(g) $\frac{12a^2b + 6ab^2}{3ab}$

Answer: _____

(h) $\frac{8a + 2b}{2ab}$

Answer: _____

(i) $\frac{6(a + b)}{3(a + b)}$

Answer: _____

(j) $\frac{8p - 16q}{4}$

Answer: _____

(k) $\frac{ab + c}{abc}$

Answer: _____

2. By first factorizing, simplify the following:

(a) $\frac{x^2 - 2x - 3}{x^2 - x - 2}$

Answer: _____

(b) $\frac{x^2 + x - 2}{x^2 - x - 6}$

Answer: _____

(c) $\frac{x^2 - 2x + 1}{x^2 - 1}$

Answer: _____

(d) $\frac{x^2 - 100}{x + 10}$

Answer: _____

(e) $\frac{x + 3}{x^2 + 6x + 9}$

Answer: _____

(f) $\frac{x^2 + 3x - 18}{x^2 - 4x + 3}$

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.
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TOPIC 2: SUMMARY



This summarizes some of the important ideas and concepts to remember.

- The **factors of a number** are all those numbers that can divide evenly into the number with no remainder.
- The **greatest common factor (GCF) of two or more numbers** is the largest number that divides exactly into two or more numbers.
- The **greatest common factor (GCF) of an algebraic expression** is the largest monomial term which will divide evenly into all the terms in the expression.
- **Factorization** is the method of expressing a number as a product of its factors.
- An algebraic expression in which the highest exponent is 2 is referred to as a **Quadratic expression or second degree expression**.
- **To factorize quadratic expression with common monomial factor**, rewrite the expression as a product of the GCF (greatest common monomial factor) and the remaining terms of the expression.
- **To factor quadratic trinomials if the coefficient of x^2 is 1**, then in general

$x^2 + bx + c = (x + n)(x + m)$ where n and m multiply to give c and add to give b .

- The **factors of the difference of two square terms** are the sum and difference of their square roots. In general

$$a^2 - b^2 = (a + b)(a - b)$$

- **To check if the answer in factorization is correct**, expand the brackets or parentheses and see if the result is the same as the original expression. If this is not then a mistake has been done.
- **Quotients** in Algebra are often called **algebraic fractions** or **rational expressions**. They are expressions involving divisions.
- The numerator and the denominator of an algebraic fraction are called its **terms**.
- **To factorize an algebraic fraction** determine the common factor of the numerator and the denominator. Then divide each by the common factor.
- When the numerator and the denominator is made up of a sum, then if every term has a common factor, divide every term by it.

REVISE LESSONS 7 – 11. THEN DO TOPIC TEST 2 IN ASSIGNMENT BOOK 2

ANSWERS TO PRACTICE EXERCISES 7-11**Practice Exercise 7**

1. (a) 1, 2, 4, 8, 16 (b) 1, 2, 4, 5, 8, 10, 20, 40 (c) 1, 17
2. (a) 5, x (c) 2, 4, x, y
(b) 2, 3, 4, 6, 12 a (d) 2, 7, 14, a, b, c
3. (a) 2 (b) 8 (c) 9 (d) 4
(e) 3 (f) m (g) a (h) 10x
4. (a) $4(a + 3b - 4c)$ (e) $2x^2(2x + 3x^2 + 6)$
(b) $6(3x - 2y - 5z)$ (f) $3b^2(7ab + 8z)$
(c) $x^2(13 - 5x + 8x^5)$ (g) $7x^2(2x^3 - 9x^2 + 5x + 3)$
(d) $ab(9ab + 11b^2 + 2a^3)$ (h) $8a^2b^2(2bc + 4ad - 5b^2e)$

Practice Exercise 8

1. (a) $c(c + 1)$ (f) $m(m + b)$
(b) $x(12x - 1)$ (g) $s(s - 2t + 3)$
(c) $5a(a - 1)$ (h) $6a(a + 2b + c)$
(d) $5p(p + 2)$ (i) $7p(2p - 4q - 5r)$
(e) $4x(2x - 3)$ (j) $7f(-f - 2gh + 1)$

Practice Exercise 9

1. (b)

X	a	-7
a	a^2	$-7a$
-9	$-9a$	63

The total for this table is $a^2 - 16a + 63$.

- (c)

X	m	-7
m	m^2	$-7m$
2	$2m$	-14

The total for this table is $m^2 - 5m - 14$.

2. (a) $x^2 + 7x + 12 = (x + 4)(\underline{x + 3})$
(b) $x^2 + \underline{7x} + 10 = (x + 5)(x + 2)$
(c) $x^2 + 6x + \underline{8} = (x + 4)(x + 2)$
(d) $a^2 + \underline{6a} + 5 = (a + 1)(\underline{a + 5})$
(e) $c^2 - 10c - \underline{11} = (c + 1)(c - 11)$

3. (a) $(n + 2)(n + 6)$ (f) $(x + 2)(x + 7)$
(b) $(x + 3)(x + 3)$ (g) $(t - 2)(t - 3)$
(c) $(c - 1)(c - 8)$ (h) $(x + 4)(x - 5)$
(d) $(y + 2)(y - 9)$ (i) $(w - 3)(w - 5)$
(e) $(r + 1)(r + 11)$ (j) $(z - 2)(z + 12)$
4. $(k - 2)(k + 7)$
-

Practice Exercise 10

1. (a) -8 and +8; $n^2 - 64 = (n + 8)(n - 8)$
(b) +11 and -11; $w^2 - 121 = (w + 11)(w - 11)$
2. (a) $(x + 6)(x - 6)$ (d) $(9 + c)(9 - c)$
(f) $(y + 7)(y - 7)$ (e) $(2x + 5)(2x - 5)$
(g) $(p + 8)(p - 8)$
3. $5(b + 1)(b - 1)$
-

Practice Exercise 11

1. (a) $x + 1$ (f) $2(2a + b)$
(b) $\frac{x}{2x + 3}$ (g) $\frac{4a + b}{ab}$
(c) $\frac{4x - 6y + 7z}{2y}$ (h) 2
(d) 5 (i) $2p - 4q$
(e) $\frac{2}{3}$ (j) $\frac{ab + c}{abc}$ not factorable, no common factor
2. (a) $\frac{x - 3}{x - 2}$ (d) $x - 10$
(b) $\frac{x - 1}{x - 3}$ (e) $\frac{1}{x + 3}$
(c) $\frac{x - 1}{x + 1}$ (f) $\frac{x + 6}{x - 1}$
-

END OF TOPIC 2

TOPIC 3

POWERS/INDICES

Lesson 12: Positive Powers

Lesson 13: Zero and Negative Powers

Lesson 14: Fractional Indices

Lesson 15: Scientific Notations

**Lesson 16: Addition and Subtraction of
Numbers in Scientific Notations**

**Lesson 17: Multiplication and Division of
Numbers in Scientific Notations**

TOPIC 3: POWERS/INDICES

Introduction



The processes of raising a quantity or a number to a power and extracting the roots of a given quantity or number are very important in applying algebra to higher mathematics.

Hence, we are to deal with them thoroughly in this topic.

Indices are a useful way of expressing large numbers. They also present us with many useful properties for manipulating them using what we called the **Law of Indices**.

The manipulation of **powers**, or **indices** or **exponents** is a very crucial underlying skill to have in algebra. Exponents play a large role in mathematical calculations.

This topic provides the meaning of exponents and the calculations associated with them. Since exponents are used abundantly in all of mathematics, the basics taught in this topic will become important building blocks for future knowledge.

In Lesson 12, you will revise both the meaning of a positive power and the rules we use when dealing with expressions with positive powers.

You will learn the three laws of indices and from those we can derive three other interesting and useful rules covered in Lessons 13 and 14 for working out or simplifying expressions with zero, negative and fractional exponents or indices.

Further new ideas about scientific notation are introduced in Lessons 15 and 16. You will revise using scientific notation in writing very large and small numbers in shorter forms.

Lesson 12: Positive Powers/Indices



You have learned about powers and exponents in Grades 8 and 9 Mathematics.



In this lesson, you will:

- revise the meaning of power
- apply rules used in simplifying and evaluating expressions containing positive powers or indices

Earlier in your study of Algebra, you learnt how to use powers or indices accurately.

First in this lesson, we will revise the meaning of powers or indices or exponents. Either name can be used. And these terms mean the same thing.

When we multiply a number by itself, we use **powers** or **indices** as they are called.

For example, the product $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ is usually written as 243.

As you can see, the product contains the same number **3**. There are five 3's multiplied together. To write products which have the same number, multiplied by itself, a certain number of time, we use the "**power notation**".

Power notation is a shorthand way of writing multiplications of the same numbers.

Consider the product; $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

We write this as 3^5 , read as "3 to the power of 5":

So, $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^5$

The number **5** is called the **index** or **power**, the number **3** is called the **base**.

An index or power or exponent is used to show that a quantity is repeatedly multiplied by itself.

Here is another product: $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$

Notice that the product contains the same number $\frac{1}{3}$. There are six $\frac{1}{3}$'s multiplied together, so the power or index is 6.

Hence, we have: $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \left(\frac{1}{3}\right)^6$ ← this read as " $\frac{1}{3}$ to the power of 6".

Products of different numbers can also be written in power notation.

For example: $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 = 5^6 \cdot 8^5$

This is read as: “5 to the power of 6 multiplied by 8 to the power of 5.”

Note: If a number occurs only once in a product then its power is 1.

For example, since 7 occurs only once in the product $2 \cdot 2 \cdot 2 \cdot 2 \cdot 7$ then the power of 7 is 1.

In power notation, $7 = 7^1$

Any single number when written in power notation always has a **power** or an **exponent** of 1.

Notes:

- When a number has a power of 2, we say the number is **squared**.

Example: 9^2 is read as “9 squared”.

- When a number has a power of 3, we say the number is **cubed**.

Example: 6^3 is read as “6 cubed”.

The power notation also applies with letters which are multiplied by them.

Examples $x = x^1$
 $m \cdot m \cdot m \cdot m \cdot m = m^5$

Products of different letters and different letters combined with numbers can also be written in power notation.

Examples $a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b \cdot c \cdot c = a^3b^4c^2$
 $2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x = 2^3x^3$ which we will see shortly can be written $8x^3$

To proceed further, we need rules to operate numbers involving multiplication of indices.

Power Rule 1 Multiplying Powers of the Same Base

Suppose we have x^3 and we want to multiply it by x^4 . That is

$$(x^3)(x^4) = (x \cdot x \cdot x)(x \cdot x \cdot x \cdot x) = x^7$$

Altogether there are seven x's multiplied together. Clearly, this is the same as x^7 . This suggests the first rule.

The first rule tells us that if we are multiplying expressions such as these, then we add the indices together. So, if we have

$$(a^m)(a^n)$$

We add the indices to get

$$(a^m)(a^n) = a^{m+n}$$

Hence the first rule states:

To multiply powers of the same base, add the exponents or indices.

Examples

$$\begin{aligned} 1. \quad (2^5)(2^2)(4^2)(4^3) &= (2^{5+2})(4^{2+3}) \\ &= (2^7)(4^5) \end{aligned}$$

$$\begin{aligned} 2. \quad (a^2)(a^4)(b^3)(b^5) &= (a^{2+4})(b^{3+5}) \\ &= a^6b^8 \end{aligned}$$

$$\begin{aligned} 3. \quad (6b^2)(3b^4) &= (6)(3)(b^2)(b^4) \\ &= 18b^{2+4} \\ &= 18b^6 \end{aligned}$$

$$\begin{aligned} 4. \quad (3xy^4)(5x^3y^3) &= (3)(5)(x)(x^3)(y^4)(y^3) \\ &= 15x^{1+3}y^{4+3} \\ &= 15x^4y^7 \end{aligned}$$

Only powers of the same base can be combined.

Power Rule 2 Power of a Power

Suppose we had a^4 and we want to raise it all to the power of 5.

That is $(a^4)^5$

That means $(a^4)(a^4)(a^4)(a^4)(a^4)$

Now the first rule tells that we should add the indices together. So that is

$$a^{4+4+4+4+4} = a^{20}$$

But note also that 20 is $(4)(5)$.

This suggests that if we have a^m all raised to the power n the result is obtained by multiplying the two powers to get $a^{(m)(n)}$, or simply a^{mn} .

We multiply the indices to get

$$(a^m)^n = a^{mn}$$

Hence the second rule states:

For a power of a power, multiply the indices or exponents.

Examples:

$$\begin{aligned} 1. \quad (x^4)^3 &= (x^4)(x^4)(x^4) \\ &= x^{4+4+4} \\ &= x^{4(3)} \\ &= x^{12} \end{aligned}$$

$$\text{Therefore, } (x^4)^3 = x^{12}$$

$$\begin{aligned} 2. \quad (6^2)^4 &= (6^2)(6^2)(6^2)(6^2) \\ &= 6^{2+2+2+2} \\ &= 6^{2(4)} \\ &= 6^8 \end{aligned}$$

$$\text{Therefore, } (6^2)^4 = 6^8$$

$$\begin{aligned} 3. \quad (2x)^6 &= (2)(2)(2)(2)(2)(2)(x)(x)(x)(x)(x)(x) \\ &= 2^{(1)(6)}x^{(1)(6)} \\ &= 2^6x^6 \\ &= 64x^6 \end{aligned}$$

$$\text{Therefore, } (2x)^6 = 64x^6$$

$$\begin{aligned} 4. \quad (3y^2)^3 &= (3)(3)(3)(y^2)(y^2)(y^2) \\ &= (3)^3y^{2(3)} \\ &= 27y^6 \end{aligned}$$

$$\text{Therefore, } (3y^2)^3 = 27y^6$$

$$\begin{aligned} 5. \quad (5m^3n^2)^3 &= 5^3(m^3)^3(n^2)^3 \\ &= 125m^9n^6 \end{aligned}$$

As you can see in the examples, the power of every term in the parentheses is multiplied by the power outside the bracket.

Power rule 3 Dividing Powers of the Same Base

Consider dividing 4^5 by 4^3 .

$$\frac{4^5}{4^3} = \frac{(4)(4)(4)(4)(4)}{(4)(4)(4)}$$

We can now begin dividing out (cancel) the common factors of 4. Three of the 4's at the top and three 4's at the bottom can be cancelled out so we are now left with

$$\frac{(4)(4)}{1} \text{ or } (4)(4)$$

This is also the same as: $\frac{4^2}{1} = 4^2$

$$\text{Therefore, } \frac{4^5}{4^3} = 4^2$$

Now let us consider dividing a^9 by a^7 .

$$\frac{a^9}{a^7} = \frac{(a)(a)(a)(a)(a)(a)(a)(a)(a)}{(a)(a)(a)(a)(a)(a)(a)}$$

$$= \frac{(a)(a)}{1}$$

$$= \frac{a^2}{1} = a^2$$

$$\text{Therefore, } \frac{a^9}{a^7} = a^2$$

Listing the results of the two examples above, we have:

$$\frac{4^5}{4^3} = 4^2 \text{ and } \frac{a^9}{a^7} = a^2$$

Can you see a rule for writing down the exponent or power of the answer?

You should be able to see in each example, that the exponent or power of the answer is equal to the difference obtained by subtracting the exponent or power in the denominator from the exponent or power in the numerator.

$$\text{That is, } \frac{4^5}{4^3} = 4^{5-3} = 4^2 \text{ and } \frac{a^9}{a^7} = a^{9-7} = a^2$$

This suggests our third rule, that

$\frac{a^m}{a^n} = a^{m-n}$

Hence the third rule states:

When dividing powers of the same base, subtract the indices or exponents.

Examples

$$1. \quad \frac{n^6}{n^3} = n^{6-3} = n^3$$

$$2. \quad \frac{24a^6}{6a^4} = 4a^{6-4} = 4a^2$$

$$3. \quad \frac{a^5b^4}{a^2b^3} = a^{5-2}b^{4-3} = a^3b^1 \text{ or } a^3b$$

$$4. \quad \frac{24p^6q^3}{6q^2} = 4p^6q^{3-2} = 4p^6q$$

$$5. \quad \frac{12x^4y^7}{6x^3} = 2x^{4-3}y^7 = 2xy^7$$

NOW DO PRACTICE EXERCISE 12

**Practice Exercise 12**

1. Rewrite the following using power notation:

- (a) $(5)(5)$
 - (b) $(3)(3)(3)(3)(3)(3)$
 - (c) $(6)(6)(x)(x)(x)(x)(x)$
 - (d) $(m)(m)(m)(m)(m)(n)(n)(n)$
 - (e) $(4)(4)(4)(a)(a)(a)(a)(b)(b)(b)(c)(c)(d)$
-

2. Find the numerical value of the following:

- (a) 4^1
 - (b) 9^3
 - (c) 2^6
 - (d) 1^7
 - (e) $(2^4)(3)^2(10)^2$
-

3. Simplify the following products:

- (a) $(3a^2)(4a^4)$
 - (b) $(6x^4y^2)(3x^5y^4)$
 - (c) $(6p^2q^4)(2pq)(4p)$
 - (d) $(7a^3b^4)(2a)(3b^2)$
 - (e) $(8m^2n^4)(4m^5n^6)$
-

4. Simplify the following divisions and write the answer in index notation.

- (a) $\frac{4^3}{4^1}$
- (b) $\frac{a^{12}}{a^8}$
- (c) $\frac{9^6 x}{9^4}$

$$(d) \frac{3^5 x^4 y^8}{3^3 x^2 y^3}$$

$$(e) \frac{18a^7 b^2}{6a^2 b^2}$$

$$(f) \frac{8^4 m^6 n^3}{8^2 m^2 n^4}$$

5. Simplify the following expressions:

$$(a) (x^3)^5$$

$$(b) (2x^3)^2$$

$$(c) (mn^4)^3$$

$$(d) (3x^2 y^3)^2$$

$$(e) (2p^4 q^2)^4$$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.
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Lesson 13: Zero and Negative Powers



In this lesson, you will:

- identify zero and negative exponents
- use zero and negative indices.

In the previous lesson, you have learned how to use positive exponents accurately. In this lesson, we shall meet exponents of different kinds, and explain their meanings and uses.

Below is the summary of the various laws of indices when ($a \neq 0$).

For Multiplication: $a^m \cdot a^n = a^{m+n}$

For Division: $a^m \div a^n = a^{m-n}$

For Involution: $(a^m)^n = a^{mn}$

For Powers of Products: $(ab)^n = a^n b^n$.

Involution (in Mathematics) is an algebraic operation in which a number, variable, expression, etc. is raised to a specified power.

$$\text{Thus } (ab)^3 = (ab)(ab)(ab) = (aaa)(bbb) = a^3 b^3$$

For Powers of Quotients: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Note that the exponents to which we have applied these laws in the past were always positive integers and the results of their application were also positive integral exponents.

The zero exponent

If we extend the meaning of powers by assuming that $a^m \div a^n = a^{m-n}$ is true for all cases, provided $a \neq 0$, it must also be true when $m = n$.

Thus we have, by the law of exponents in division: $\frac{a^n}{a^n} = a^{n-n} = a^0$

But if we divide the numerator by the denominator, $\frac{a^n}{a^n} = 1$.

Therefore $a^0 = 1$.

Any quantity (except 0) with zero exponents has a numerical value of 1.

Examples

1. $x^0 = 1$
2. $7^0 = 1$
3. $[3x(2a - b)^3]^0 = 1$
4. $5a^0 = 5$

We have therefore, shown that the stated law hold if $m = n$; and we obtain a definite value of any number (except 0) to the zero power.

The meaning of a^0

You should notice that a^0 is merely a symbol for $\frac{a^n}{a^n}$. In other words, it means that some power of a has been divided by the same power of a .

$$\text{For example, } 3^0 = \frac{3^4}{3^4} = \frac{3^6}{3^6} = \frac{3^m}{3^m} = \frac{3^n}{3^n} = 1$$

The negative exponent

We will now finally look at numbers or terms that have negative powers or exponents.

For examples: 4^{-5} , $2a^{-3}b^{-2}$, $5^{-3}p^{-4}$ and y^{-2} all have negative powers.

What does negative power means?

By the law of exponents in division, $\frac{a^0}{a^n} = a^{0-n} = a^{-n}$

But also $\frac{a^0}{a^n} = \frac{1}{a^n}$, since a^0 has already been proven equal to 1.

Therefore, $a^{-n} = \frac{1}{a^n}$, since both are equal to the same thing.

$$\text{Also } \frac{1}{a^{-n}} = \frac{1}{\frac{1}{a^n}} = \cancel{1} \left(\frac{a^n}{\cancel{1}} \right) = a^n$$

Hence:

Any quantity with a negative exponent is equal to the reciprocal of that quantity with corresponding positive exponent.

From the above principle, it also follows that, any factor of the numerator of a fraction may be transferred to the denominator, or any factor of the denominator may be transferred to the numerator, if the sign of its exponent is changed.

Examples

$$1. \quad 2^{-3} = \frac{1}{2^3}$$

$$5. \quad 2a^{-2}b^{-3} = \frac{2}{a^2b^3}$$

$$3. \quad \frac{a^2b^{-3}}{x^{-2}y} = \frac{a^2x^2}{b^3y}$$

$$4. \quad (3^0)(5p^{-7}q^{-4}) = (3^0)(5)\left(\frac{1}{p^7}\right)\left(\frac{1}{q^4}\right)$$

$$\begin{aligned} 5. \quad \frac{a^0b^2c^3}{b^{-2}c^{-4}} &= a^0b^2c^3(b^2)(c^4) \\ &= 1b^4c^7 \\ &= b^4c^7 \end{aligned}$$

NOW DO PRACTICE EXERCISE 13

**Practice Exercise 13**

1. Evaluate each of the following:

(a) 4^0

(b) $2b^0$

(c) $\left(\frac{1}{2}\right)^0$

(d) $6 \cdot 7^0$

(e) $5^0 \cdot 8^0 \cdot 4^2$

2. Evaluate each of the following leaving your answer as a proper fraction.

(a) 2^{-9}

(b) 3^{-5}

(c) 5^{-3}

(d) 7^{-3}

(e) 8^{-3}

3. Rewrite with only positive powers simplifying as far as possible.

(a) s^{-7}

(h) $5 \cdot x^{-3}$

(i) $3^0 \cdot r^{-6}$

(j) $b^{-2} \cdot c^{-4}$

(k) $m^{-7}n^{-4}$

(l) $2^{-6}e^{-7}f^{-4}$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.
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Lesson 14: Fractional Indices



You learnt to use zero and negative indices in the previous lesson.



In this lesson, you will:

- identify fractional indices
- use fractional indices

So far we have dealt with numbers and integers with both positive and negative powers. What will we do if we had a fraction for power, like $a^{\frac{1}{2}}$?

To see how to deal with fractional powers consider the following:

Suppose we have identical numbers multiplying together to give another number, as in, for example

$$(9)(9) = 81$$

Then we know that 9 is a square root of 81.

$$\text{That is, if } 9^2 = 81, \text{ then } 9 = \sqrt{81}$$

Now suppose we found that

$$(a^p)(a^p) = a$$

That is, when we multiplied a^p by itself we got the result a . This means that a^p must be a square root of a .

However, look at this another way: noting that $a = a^1$, and also that, from the first rule, $(a^p)(a^p) = a^{2p}$ we see that if $(a^p)(a^p) = a$ then

$$a^{2p} = a^1$$

from which

$$2p = 1$$

and so

$$p = \frac{1}{2}$$

This shows that $a^{\frac{1}{2}}$ must be the square root of a . That is

The power $\frac{1}{2}$ denotes a square root: $a^{\frac{1}{2}} = \sqrt{a}$
--

Similarly $a^{\frac{1}{3}} = \sqrt[3]{a}$ this is the cube root of a

and $a^{\frac{1}{4}} = \sqrt[4]{a}$ this is the fourth root of a

More generally

$$a^{\frac{1}{q}} = \sqrt[q]{a}$$

In the same way, if we represent $\sqrt[3]{a}$ as a^q , then

$$(\sqrt[3]{a})(\sqrt[3]{a})(\sqrt[3]{a}) = a$$

$$(a^q)(a^q)(a^q) = a$$

$$a^{q+q+q} = a^1$$

$$a^{3q} = a^1$$

Powers of the same base

$$3q = 1$$

$$q = \frac{1}{3}$$

that is, $a^q = a^{\frac{1}{3}}$

Therefore, $\sqrt[3]{a} = a^{\frac{1}{3}}$.

In general, we define

$$\sqrt[q]{a} = a^{\frac{1}{q}}$$

Now let us work through the following examples:

Example 1

What do we mean by $16^{\frac{1}{4}}$?

Solution:

For this we need to know what number when multiplied together four times gives 16.

The answer is 2.

$$\text{So, } 16^{\frac{1}{4}} = \sqrt[4]{16} = 2$$

$$\text{So, } 16^{\frac{1}{2}}$$

Example 2 Evaluate: $81^{\frac{1}{2}}$

Solution:

For this we need to know what number when multiplied by itself gives 81. The answer is 9.

$$\text{So, } 81^{\frac{1}{2}} = \sqrt{81} = 9$$

Example 3 Evaluate: $243^{\frac{1}{5}}$

Solution: For this we need to know what number when multiplied together five times gives us 243.

If we are familiar with the times table, we might spot that $243 = 3 \times 81$, and also that $81 = 9 \times 9$.

$$\text{So, } 243^{\frac{1}{5}} = [(3)(81)]^{\frac{1}{5}} = [(3)(9)(9)]^{\frac{1}{5}} = [(3)(3)(3)(3)(3)]^{\frac{1}{5}}$$

So 3 multiplied by itself five times equals 243.

$$\text{Hence, } 243^{\frac{1}{5}} = 3$$

Notice that in doing this, it is important to be able to recognize what factors numbers are made up of. For example, it is important to be able to recognize that:

$$16 = 2^4, \quad 16 = 4^2, \quad 81 = 9^2, \quad 81 = 3^4 \text{ and so on}$$

You will find calculations much easier if you can recognize in numbers their composition as powers of simple numbers such as 2, 3, 4, and 5. Once you have got this firmly fixed in your mind, this sort of calculation becomes straightforward.

Now look at the next example.

Example 4 Evaluate: $\sqrt[3]{x^2}$

$$\begin{aligned} \text{Solution: } \sqrt[3]{x^2} &= (x^2)^{\frac{1}{3}} \\ &= x^{2(\frac{1}{3})} \\ &= x^{\frac{2}{3}} \end{aligned}$$

This leads to the following rule:

$\sqrt[q]{a^p} = a^{\frac{p}{q}}$

Using this rule, let us evaluate each of the following examples.

Example 5 Evaluate: $(8)^{\frac{2}{3}}$

$$\begin{aligned}\text{Solution: } (8)^{\frac{2}{3}} &= (2^3)^{\frac{2}{3}} \\ &= 2^{3(\frac{2}{3})} \\ &= 2^2 \\ &= 4\end{aligned}$$

$$\begin{aligned}\text{For an alternative solution: } (8)^{\frac{2}{3}} &= (\sqrt[3]{8})^2 \\ &= 2^2 \\ &= 4\end{aligned}$$

Example 6 Evaluate: $9^{\frac{3}{2}}$

$$\begin{aligned}\text{Solution: } 9^{\frac{3}{2}} &= (3^2)^{\frac{3}{2}} \\ &= 3^{2(\frac{3}{2})} \\ &= 3^3 \\ &= 27\end{aligned}$$

$$\begin{aligned}\text{For an alternative solution: } (9)^{\frac{3}{2}} &= (\sqrt{9})^3 \\ &= 3^3 \\ &= 27\end{aligned}$$

Example 7 Evaluate: $64^{\frac{2}{3}}$

$$\begin{aligned}\text{Solution: } 64^{\frac{2}{3}} &= (4^3)^{\frac{2}{3}} \\ &= 4^{3(\frac{2}{3})} \\ &= 4^2 \\ &= 16\end{aligned}$$

$$\begin{aligned}\text{For an alternative solution: } (64)^{\frac{2}{3}} &= (\sqrt[3]{64})^2 \\ &= 4^2 \\ &= 16\end{aligned}$$

NOW DO PRACTICE EXERCISE 14

**Practice Exercise 14**

1. Replace each radical by an equivalent fractional exponents

a. $\sqrt{5}$

b. $\sqrt{7}$

c. $\sqrt[3]{x}$

d. $\sqrt[4]{m}$

e. $\sqrt[4]{c^3}$

2. Rewrite each expression in radical form.

a. $8^{\frac{1}{2}}$

b. $5^{\frac{1}{3}}$

c. $y^{\frac{3}{5}}$

d. $x^{\frac{2}{3}}$

e. $b^{\frac{m}{n}}$

3. Evaluate each of the following expression.

a. $4^{-\frac{5}{2}}$

b. $16^{\frac{3}{4}}$

c. $625^{\frac{1}{4}}$

d. $27^{\frac{4}{3}}$

e. $729^{\frac{1}{3}}$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3
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Lesson 15: Scientific Notations



You have learned to use zero and negative indices in the previous lesson.



In this lesson, you will:

- discuss scientific notation
- use scientific notation to write very large number in a much shorter form
- change numbers written in scientific notation into ordinary notation.

Scientific notation is the way that scientists easily handle very large numbers, for example 59 000 000 000 or very small numbers, for example 0.0000000056, in a much shorter forms.

The word “scientific” is used often when extremely large numbers occur in scientific measurements. For example, the speed of light is 300 000 000 km per hour. Similarly very, very small numbers frequently occur in scientific measurements. For example, the charge on a proton (an atomic particle) is only 0.00000000000000000016 coulombs.

When using scientific notation to distinguish between large numbers and small numbers we say that a “large number” is greater than or equal to 1 and a “small number” is less than one, that is, a small number is always a decimal.

In order to use scientific notation, you must be able to write powers of 10 in index form. For example:

$1 = 10^0$	$1 = 1 \times 10^0$
$10 = 1 \times 10^1$	$\frac{1}{10} = 0.1 = 1 \times 10^{-1}$
$100 = 1 \times 10^2$	$\frac{1}{100} = 0.01 = 1 \times 10^{-2}$
$1000 = 1 \times 10^3$	$\frac{1}{1000} = 0.001 = 1 \times 10^{-3}$
$10000 = 1 \times 10^4$	$\frac{1}{10\,000} = 0.0001 = 1 \times 10^{-4}$
$100\,000 = 1 \times 10^5$	$\frac{1}{100\,000} = 0.00001 = 1 \times 10^{-5}$
$1\,000\,000 = 1 \times 10^6$	$\frac{1}{1\,000\,000} = 0.000001 = 1 \times 10^{-6}$

A number is in scientific notation when it is written as

$A \times 10^n$

where **A** is a number between 1 and 10 and **n** is an integer.

- For any number greater than 1, **n** is positive.
- For any number less than 1, **n** is negative.

Scientific notation for large numbers

Example 1

Let us look at how to write 658 for example, in scientific notation.

First, rewrite the number so that it becomes a number in the range 1 to 10 by simply placing a decimal point after the first digit. In this example, we have 6 as the first digit.

Hence, $658 \rightarrow 6.58$

Now to make 6.58 equal to the original number (658) we need to multiply 6.58 by 100.

$$6.58 \times 100 = 658$$

The number 100 in the product has to be written as a power of 10.

Now, $100 = 10 \times 10 = 10^2$ so, 100 written as a power of 10 is 10^2 .

Hence we have, $6.58 \times 100 = 6.58 \times 10^2$

Therefore, in scientific notation $658 = 6.58 \times 10^2$

Example 2

Write the number 5 000 000 in scientific notation.

Solution:

Rewriting the number to be in the range of 1 to 10:

$$5\,000\,000 \rightarrow 5$$

(Notice we do not write in the decimal point or the 0's after the decimal point in this case since **$5.000000 = 5.0 = 5$**)

Now we multiply 5 by 1 000 000 to make it equal to 5 000 000:

$$5 \times 1\,000\,000 = 5\,000\,000$$

The number 1 000 000 in the product $5 \times 1\,000\,000$ must be written as a power of 10:

$$1\,000\,000 = (10)(10)(10)(10)(10)(10) = 10^6$$

$$\text{So, } 5 \times 1\,000\,000 = 5 \times 10^6$$

Therefore, $5\,000\,000 = 5 \times 10^6$

Example 3

Write the number 2345.8 in scientific notation.

Solution:

Change the number 2345.8 into a number between 1 and 10. This is done by writing a decimal point after the first digit which is 2.

$$2345.8 \text{ -----} \rightarrow 2.3458$$

Now to make 2.3458 equal to the original number (2345.8), we need to multiply 2.3458 by 1000 since:

$$2.3458 \times 1000 = 2345.8$$

The number 1000 thousand in the product 2.3458×1000 must be written as a power of 10.

So, $1000 = 10 \times 10 \times 10 = 10^3$

Giving $2.3458 \times 1000 = 2.3458 \times 10^3$

Therefore, $2345.8 = 2.3458 \times 10^3$

Example 4

Write 75 000 000 in scientific notation.

Solution: $75\,000\,000 = 7.5 \times 10\,000\,000$
 $= 7.5 \times 10^7$

Example 5

Write 6 179 500 in scientific notation.

Solution: $6\,179\,500 = 6.1795 \times 1\,000\,000$
 $= 6.1795 \times 10^6$

Example 6

Write 255 456 032 in scientific notation.

Solution: $255\,456\,032 = 2.55456032 \times 100\,000\,000$
 $= 2.55456032 \times 10^8$

Scientific notation for small numbers

As with large numbers, when a small number is written in its equivalent scientific notation, it is written as a number from 1 up to but not including 10, multiplied by a power of 10.

For example, instead of writing 0.0000000056, we write 5.6×10^{-9} . So, how does this work?

Look at the following examples:

Example 7

Write 0.0057 in scientific notation.

Solution:

First, rewrite or change the number 0.0057 so that it becomes a number between 1 and 10 by moving the decimal point and writing it after the first non-zero digit, that is, after the digit 5. We do not write the 0's which were in front of this digit as this is unnecessary.

So we have $0.0057 \rightarrow 5.7$

Now to make 5.7 equal to the original number (0.0057), we must divide it by 1000.

$$\text{Thus } \frac{5.7}{1000} = 5.7 \times \frac{1}{1000}$$

Since we want the number to be in scientific notation, the fraction $\frac{1}{1000}$ in the product $5.7 \times \frac{1}{1000}$ must be written as a power of 10.

$$\text{Now } \frac{1}{1000} = \frac{1}{10 \times 10 \times 10}$$

$$= \frac{1}{10^3}$$

$$= 10^{-3} \quad (\text{Remember the rule } \frac{1}{a^n} = a^{-n} \text{ in Lesson 13})$$

$$\text{Hence, we have } 5.7 \times \frac{1}{1000} = 5.7 \times 10^{-3}$$

Therefore, in scientific notation: $0.0057 = 5.7 \times 10^{-3}$.

Notice that the power of 10 is negative. This is always true when numbers less than 1 are written in scientific notation.

Now look at the next example.

Example 8 Write in scientific notation: 0.000467

Solution:

First, rewrite or change the number 0.000467 so that it becomes a number between 1 and 10 by moving the decimal point and writing it after the first non-zero digit, that is, after the digit 4. We do not write the 0's which were in front of this digit as this is unnecessary.

So we have $0.000467 \rightarrow 4.67$

To make 4.67 equal to the original number (0.000467) we must divide it by 10 000.

$$\text{Thus } \frac{4.67}{10\,000} = 4.67 \times \frac{1}{10\,000}$$

Since we want the number to be in scientific notation, the fraction $\frac{1}{10\,000}$ in the product $4.67 \times \frac{1}{10\,000}$ must be written as a power of 10.

$$\begin{aligned} \text{Now } \frac{1}{10\,000} &= \frac{1}{10 \times 10 \times 10 \times 10} \\ &= \frac{1}{10^4} \\ &= 10^{-4} \quad (\text{Remember the rule } \frac{1}{a^n} = a^{-n} \text{ in Lesson 13}) \end{aligned}$$

$$\text{Hence, we have } 4.67 \times \frac{1}{10\,000} = 4.67 \times 10^{-4}$$

Therefore, in scientific notation: $0.000467 = 4.67 \times 10^{-4}$.

Example 9 Write in scientific notation: 0.00615478

$$\begin{aligned} \text{Solution: } 0.00615478 &= 6.15478 \times \frac{1}{1000} \\ &= 6.15478 \times 10^{-3} \end{aligned}$$

Example 10 Write in scientific notation: 0.0000000089

$$\begin{aligned} \text{Solution: } 0.0000000089 &= 8.9 \times \frac{1}{1\,000\,000\,000} \\ &= 8.9 \times 10^{-9} \end{aligned}$$

Now, look at the table below where we have summarized the answers to all the 9 examples we have discussed so far.

Original Number	Number when in scientific notation	Number of places the decimal point is moved to the left or right
658	6.58×10^2	2 to the left
5 000 000	5×10^6	6 to the left
2345.8	2.3458×10^3	3 to the left
75 000 000	7.5×10^7	7 to the left
255 456 032	2.55456032×10^8	8 to the left
0.0057	5.7×10^{-3}	3 to the right
0.000467	4.67×10^{-4}	4 to the right
0.00615478	6.15478×10^{-3}	3 to the right
0.0000000089	8.9×10^{-9}	9 to the right

Looking at the pattern connecting the number of places moved to the left or to the right and the corresponding power of 10, we can summarize the steps in writing large and small numbers into its equivalent scientific notation.

When writing numbers into its equivalent scientific notation,

1. Change the number into a number between 1 and 10.
(A)
2. Multiply it by the appropriate power of ten. (10^n)

For large number:

Step 1: Move the decimal place to the **left** to create a new number from 1 to 10.

Step 2: Determine the exponent, which is the number of times you moved the decimal point.

Step 3: Put the number in the correct form for scientific notation.

For small number:

Step 1: Move the decimal place to the **right** to create a new number from 1 to 10.

Step 2: Determine the exponent, which is the number of times you moved the decimal point.

Step 3: Put the number in the correct form for scientific notation.

Now go to the next page and look at changing numbers written in scientific notation into ordinary numbers.

Rewriting a number in scientific notation into ordinary numbers

To change a number written in scientific notation into ordinary notation, simply move the decimal point to the right or left, the number of places moved being equal to the power of 10.

- Positive exponent, move the decimal point to the right.
- Negative exponent, move the decimal point to the left.

Example 1

Write in ordinary notation the number 3.12×10^{11} .

Solution:

3.12×10^{11} ←----- The power of 10 is positive 11.

Moved the decimal point 11 places to the right.

3.12000000000 -----► 312 000 000 000

Therefore, $3.12 \times 10^{11} = 312\,000\,000\,000$

Example 2

Write in ordinary notation the number 3.1273×10^3 .

Solution:

3.12×10^3 ←----- The power of 10 is positive 3.

Moved the decimal point 3 places to the right

3.1273 -----► 3127.3

Therefore, $3.1273 \times 10^3 = 3127.3$

Example 3

Write in ordinary notation the number 5.68×10^{-5} .

Solution:

5.68×10^{-5} ←----- The power of 10 is negative 5.

Move the decimal point 5 places to the left and add leading zeros.

00005.68 -----► 0.0000568

Therefore, $5.68 \times 10^{-5} = 0.0000568$

We always use zeros as place holders.

After a little practice, you should be able to do the steps in one go.

Here are other examples.

Example 3

Write in ordinary notation the number 6.32×10^7 .

Solution:

$$6.32 \times 10^7 = 63\,200\,000$$

Example 4

Write in ordinary notation the number 9.543×10^{-4} .

Solution:

$$9.543 \times 10^{-4} = 0.0009543$$

Example 5

The human population of Australia in 2013 was about 23.13×10^6 . Write this number in ordinary notation.

Solution:

23.13×10^6 ←----- The power of 10 is positive 6.

Moved the decimal point 6 places to the right

23.130000 -----> $23\,130\,000$

Therefore, $23.13 \times 10^6 = 23\,130\,000$

NOW DO PRACTICE EXERCISE15

**Practice Exercise 15**

1. Write the following numbers in scientific notation:

- (a) 94 800
 - (b) 7000
 - (c) 60 000 000
 - (d) 544 696 404
 - (e) 9678.2
 - (f) 756 400.5
 - (g) 5145.09
-

2. Write the following numbers in scientific notation:

- (a) 0.000875
 - (b) 0.00000066
 - (c) 0.004
 - (d) 0.0596
 - (e) 0.000000000847
-

3. The recorded human population of the earth in the year 2013 was approximately 7 215 000 000 (7.215 billion). Write this figure in scientific notation.

4. The number of millimetres in 1 kilometre is 1 000 000. Write this figure in scientific notation.

Lesson 16: Addition and Subtraction of Numbers in Scientific Notation



You have learned how to use scientific notation in writing very large and very small numbers.



In this lesson, you will;

- identify the steps in adding and subtracting numbers in scientific notation.
 - add and subtract numbers expressed in scientific notation.
-

One of the properties of quantities with exponents is that numbers with exponents can be added and subtracted only when they have the same base and exponent. Since all numbers in scientific notation have the same base (10), we need only worry about the exponents.

To be added or subtracted, two numbers in scientific notation must be done so that their bases have the same exponent. This will ensure that corresponding digits in their coefficients have the same place value.

Multiplying a number by another number with the same base is equivalent to multiplying their coefficients and adding their exponents. Therefore, if we want to add two quantities written in scientific notation whose exponents do not match, we can simply write one of the powers of 10 as the product of two smaller powers of 10, one of which agrees with the other term.

Alternately, if we want to preserve the exponent of the term with the larger power of 10, we can simultaneously multiply and divide the other term by a power of 10, applying the rule for multiplication of exponents in one case and dividing the coefficient in the other. It is this procedure that we outline below.

Once the numbers have the same base and exponents, we can add or subtract their coefficients.

To add or subtract two numbers in scientific notations:

1. Determine the number by which to increase the smaller exponent so, it is equal to the larger exponent.
2. Increase the smaller exponent by this number and move the decimal point of the number with the smaller exponent to the left the same number of places. (i.e. divide by the appropriate power of 10).
3. Add or subtract the new coefficients.
4. If the answer is not in scientific notation (i.e. if the coefficient is not between 1 and 10), convert it to scientific notation.

Note: If the numbers start with the same exponents, their coefficients can be added, but be careful, the answer might need to be converted to scientific notation.

All numbers are converted to the same power of 10, and the digit terms are added or subtracted.

Example 1:

Add: $(2.456 \times 10^5) + (6.0034 \times 10^8)$

Solution:

Step 1: $8 - 5 = 3$. The smaller exponent must be increased by 3.

Step 2: $2.456 \times 10^5 = 0.002456 \times 10^8$

Step 3: $0.002456 \times 10^8 + 6.0034 \times 10^8 = 6.005856 \times 10^8$

Step 4: 6.005856×10^8 is in scientific notation.

Therefore, $2.456 \times 10^5 + 6.0034 \times 10^8 = 6.005856 \times 10^8$

Example 2

Add: $(6.7 \times 10^3) + (5.1 \times 10^4)$

Solution:

Step 1: $4 - 3 = 1$. The smaller exponent must be increased by 1.

Step 2: $6.7 \times 10^3 = 0.67 \times 10^4$

Step 3: $5.1 \times 10^4 + 0.67 \times 10^4 = 5.77 \times 10^4$

Step 4: 5.77×10^4 is in scientific notation.

Therefore, $(6.7 \times 10^3) + (5.1 \times 10^4) = 5.77 \times 10^4$

Example 3

Subtract: $(5.10802 \times 10^3) - (6.1 \times 10^{-2})$

Solution:

Step 1: $3 - (-2) = 5$. The smaller exponent must be increased by 5.

Step 2: $6.1 \times 10^{-2} = 0.000061 \times 10^3$

Step 3: $5.10802 \times 10^3 - 0.000061 \times 10^3 = 5.107959 \times 10^3$

Step 4: 5.107959×10^3 is in scientific notation.

Therefore, $5.10802 \times 10^3 - 6.1 \times 10^{-2} = 5.107959 \times 10^3$

Example 4

Subtract: $(4.801 \times 10^3) - (2.2 \times 10^7)$

Solution:

Step 1 $7 - 3 = 4$. The smaller exponent must be increased by 4.

Step 2 $4.801 \times 10^3 = 0.0004801 \times 10^7$

Step 3 $0.0004801 \times 10^7 - 2.2 \times 10^7 = -2.1995199 \times 10^7$

Step 4 -2.1995199×10^7 is in scientific notation.

Therefore, $(4.801 \times 10^3) - (2.2 \times 10^7) = -2.1995199 \times 10^7$

After little practice, you can directly add or subtract numbers in scientific notations without showing the steps like the following examples.

Example 5

Add $(4.3 \times 10^7) + (2.97 \times 10^5)$

Solution:

$$(4.3 \times 10^7) + (2.97 \times 10^5) = 430 \times 10^5 + 2.97 \times 10^5$$

$$= \mathbf{432.97 \times 10^5}$$

Example 6

Add $(4.215 \times 10^{-2}) + (3.2 \times 10^{-4})$

Solution:

$$(4.215 \times 10^{-2}) + (3.2 \times 10^{-4}) = (4.215 \times 10^{-2}) + (0.032 \times 10^{-2})$$

$$= \mathbf{4.247 \times 10^{-2}}$$

Example 7

Subtract $(1.4 \times 10^{-5}) - (5.67 \times 10^{-6})$

Solution:

$$(1.4 \times 10^{-5}) - (5.67 \times 10^{-6}) = (1.4 \times 10^{-5}) - (0.567 \times 10^{-5})$$

$$= 0.833 \times 10^{-5}$$

$$0.833 \times 10^{-5} = 8.33 \times 10^{-6} \text{ in scientific notation}$$

Therefore, $(1.4 \times 10^{-5}) - (5.67 \times 10^{-6}) = 8.33 \times 10^{-6}$

NOW DO PRACTICE EXERCISE 16

**Practice Exercise 16**

Evaluate each of the following, giving your answers in scientific notations.

1. $(2 \times 10^3) + (3.6 \times 10^4)$

6. $(2.5 \times 10^6) + (8.13 \times 10^8)$

2. $(7 \times 10^5) - (5.2 \times 10^4)$

7. $(9.67 \times 10^9) - (5.1 \times 10^6)$

3. $(4.9 \times 10^2) + (7.9 \times 10^3)$

8. $(8.28 \times 10^{-5}) - (4.1 \times 10^{-7})$

4. $(2.19 \times 10^{-7}) - (1.8 \times 10^{-8})$

9. $4.6 \times 10^2 - 2 \times 10^{-2}$

5. $(4 \times 10^3) + (6.17 \times 10^7)$

10. $(2.6 \times 10^{-1}) + (3.48 \times 10^2)$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.
--

Lesson 17: Multiplication and Division of Numbers in Scientific Notation



You learnt to add and subtract numbers in scientific notation in the last lesson.



In this lesson, you will:

- identify the steps in multiplying and dividing numbers in scientific notation
- multiply and divide numbers expressed in scientific notation.

Multiplying Numbers in Scientific Notation

Quantities with exponents can be multiplied and divided easily if they have the same base. Since all numbers in scientific notation have base 10, we can always multiply them and divide them.

Here are the steps to multiply two numbers in scientific notation:

To multiply numbers in scientific notation:

- Multiply the decimal numbers.
- Add the exponents of the powers of 10.
- Place the new power of 10 with the decimal in scientific notation form.
- If decimal number is greater than 10, count the number of times the decimal moves to the left, and add this number to the exponent.

Example 1

Multiply (2.6×10^7) by (6.3×10^4) .

Solution:

$$(2.6 \times 10^7) \text{ by } (6.3 \times 10^4)$$

Step 1: $(2.6)(6.3) = 16.38$ Multiply the decimal numbers.

Step 2: $7 + 4 = 11$ Add the exponents.

Step 3: 16.38×10^{11} Place the new power of 10 with the decimal in scientific notation form.

Step 4: 16.38×10^{11}
 $\curvearrowright$
 1.638×10^{12} The new decimal number is greater than 10, so count the number of places the decimal moves to put the number between 1 and 10. Add this number to the power of 10. In this case, the decimal point moves one place, so add 1 to the power.

Therefore, $(2.6 \times 10^7)(6.3 \times 10^4) = 1.638 \times 10^{12}$.

Example 2

Multiply (3.4×10^6) by (4.2×10^3) .

Solution:

$$(3.4 \times 10^6)(4.2 \times 10^3)$$

Step 1: $(3.4)(4.2) = 14.28$ Multiply the decimal numbers.

Step 2: $6 + 3 = 9$ Add the exponents.

Step 3: 14.28×10^9 Place the new power of 10 with the decimal in scientific notation form.

Step 4: 14.28×10^9
 $\quad \quad \quad \curvearrowright$
 $\quad \quad \quad 1.428 \times 10^{10}$ The new decimal number is greater than 10, so count the number of places the decimal moves to put the number between 1 and 10. Add this number to the power of 10. In this case, the decimal point moves one place, so add 1 to the power.

Therefore, $(3.4 \times 10^6)(4.2 \times 10^3) = 1.428 \times 10^{10}$

After little practice, you can directly multiply numbers in scientific notations without showing the steps like the following examples.

Example 3

Multiply $(6.73 \times 10^{-5})(2.91 \times 10^2)$.

Solution:

$$\begin{aligned}(6.73 \times 10^{-5})(2.91 \times 10^2) &= (6.73)(2.91) \times 10^{(-5 + 2)} \\ &= 19.58 \times 10^{-3} \\ &= 1.958 \times 10^{-2}\end{aligned}$$

Example 4

Simplify $(5.1 \times 10^4)(2.5 \times 10^3)$.

Solution:

$$\begin{aligned}(5.1 \times 10^4)(2.5 \times 10^3) &= (5.1)(2.5) \times 10^{(4 + 3)} \\ &= 12.75 \times 10^7 \\ &= 1.275 \times 10^8\end{aligned}$$

Dividing Numbers in Scientific Notation

To divide numbers in scientific notation:

1. Divide the decimal numbers.
2. Subtract the exponents of the powers of 10.
3. Place the new power of 10 with the decimal in scientific notation form.
4. If the resulting decimal number is less than 1, move the decimal point to the right and decrease the exponent by the number of places that the decimal point moved.

Example 1

Divide: $\frac{1.23 \times 10^{11}}{2.4 \times 10^4}$

Solution:

Step 1: $\frac{1.23}{2.4} = 0.5125$ Divide the decimal numbers.

Step 2: $11 - 4 = 7$ Subtract the exponents.

Step 3: 0.5125×10^7 Place the new power of 10 with the decimal in scientific notation form.

Step 4: 0.5125×10^7
 5.125×10^6 The new decimal number is not between 1 and 10, move the decimal point one place to the right and decrease the exponent by 1.

Therefore, $\frac{1.23 \times 10^{11}}{2.4 \times 10^4} = 5.125 \times 10^6.$

Example 2

Divide: $\frac{4 \times 10^4}{3.63 \times 10^{-4}}$

Solution:

Step 1: $\frac{4}{3.63} = 1.10193$ Divide the decimal numbers.

Step 2: $4 - (-4) = 8$ Subtract the exponents.

Step 3: 1.10193×10^8 Place the new power of 10 with the decimal in scientific notation form.

Step 4: 1.10193×10^8 The new decimal number is between 1 and 10.

Therefore, $\frac{4 \times 10^4}{3.63 \times 10^{-4}} = 1.10193 \times 10^8$

After little practice, you can directly divide numbers in scientific notations without showing the steps like the following examples.

Example 3

Divide: $\frac{3.5 \times 10^{-6}}{5 \times 10^{-2}}$

Solution: $\frac{3.5 \times 10^{-6}}{5 \times 10^{-2}} = \frac{3.5}{5} \times 10^{-6 - (-2)}$

$= 0.7 \times 10^{-4}$ ← This is not in proper scientific notation.

$= 7 \times 10^{-5}$ ← This is the proper scientific notation.

Example 4

Divide: $\frac{8.9 \times 10^6}{8.4 \times 10^6}$

Solution: $\frac{8.9 \times 10^6}{8.4 \times 10^6} = \frac{8.9}{8.4} \times 10^{6-6}$

$= 1.06 \times 10^0$

Example 5

Divide: $\frac{1.62 \times 10^{-6}}{5.3 \times 10^6}$

Solution: $\frac{1.62 \times 10^{-6}}{5.3 \times 10^6} = \frac{1.62}{5.3} \times 10^{-6-6}$

$= 30.57 \times 10^{-12}$ ← This is not in proper scientific notation.

$= 3.057 \times 10^{-1} \times 10^{-12}$

$= 3.057 \times 10^{-13}$ ← This is the proper scientific notation.

NOW DO PRACTICE EXERCISE 17

**Practice Exercise 17**

Simplify each of the following and write your answer in scientific notation.

1. $(1.08 \times 10^{-3})(9.3 \times 10^{-3})$

2. $(2 \times 10^{-4})(8.1 \times 10^{-1})$

3. $(2.32 \times 10^{-6})(4 \times 10^{-5})$

4. $(3.48 \times 10^3)(9.8 \times 10^4)$

5. $(7.1 \times 10^{-5})(6.7 \times 10^{-6})$

6. $\frac{7.1 \times 10^6}{8.2 \times 10^1}$

7. $\frac{4 \times 10^4}{3.63 \times 10^{-4}}$

8. $\frac{17.04 \times 10^5}{19.4 \times 10^2}$

9. $\frac{9.75 \times 10^5}{1.9 \times 10^2}$

10. $\frac{5.26 \times 10^5}{4.7 \times 10^3}$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

TOPIC 3: SUMMARY



This summarizes some of the important ideas and concepts to remember.

- **Power notation** is a shorthand way of writing multiplications of the same numbers.
- An **index or power** is used to show that a quantity is repeatedly multiplied by itself.
- Any single number when written in power notation always has a power or an exponent of 1.
- **Laws of indices**
 - Power Rule 1: To multiply powers of the same base, add the indices or exponents.

$$(a^m)(a^n) = a^{m+n}$$

- Power Rule 2: For a power of a power, multiply the exponents or indices.

$$(a^m)^n = a^{mn}$$

- Power Rule 3: When dividing powers of the same base, subtract the indices or exponents.

$$\frac{a^m}{a^n} = a^{m-n}$$

- Powers of Product Rule 4: When a product is raised to a power, we can distribute that power through to each term in the product.

$$(ab)^n = a^n b^n$$

- Powers of Quotients Rule 5: When a quotient is raised to a power, you can apply the power to the numerator and denominator individually

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

- **Zero Exponent** Rule 6: Any quantity (except 0) with zero exponents has a numerical value of 1.

$$a^0 = 1$$

- **Negative indices** Rule 7: Any quantity with a negative exponent is equal to the reciprocal of that quantity with corresponding positive exponent.

$$\left(\frac{a}{b}\right)^{-m} = \frac{a^{-m}}{b^{-m}} = \frac{b^m}{a^m}$$

- **Fractional Indices**

- The power of $\frac{1}{2}$ denotes a square root:

$$a^{\frac{1}{2}} = \sqrt{a}$$

- The power of $\frac{1}{3}$ denotes a cube root.

$$a^{\frac{1}{3}} = \sqrt[3]{a}$$

- The power of $\frac{1}{4}$ denotes a fourth root.

$$a^{\frac{1}{4}} = \sqrt[4]{a}$$

- **Scientific notation** is a method of writing or displaying numbers in terms of a decimal number between 1 and 10 multiplied by a power of 10. A number is in scientific notation when it is written as **A x 10ⁿ**.
- **When writing numbers into scientific notation**
 - e. Change the number into a number between 1 and 10. (**A**)
 - f. Multiply by the appropriate power of ten, (**10ⁿ**)
- To **add** and **subtract** numbers in scientific notations:
 1. Determine the number by which to increase the smaller exponent by so it is equal to the larger exponent.
 2. Increase the smaller exponent by this number and move the decimal point of the number with the smaller exponent to the left the same number of places. (i.e. divide by the appropriate power of 10).
 3. Add or subtract the new coefficients.
 4. If the answer is not in scientific notation (i.e. if the coefficient is not between 1 and 10) convert it to scientific notation.
- To **multiply** numbers in scientific notations:
 1. Multiply the decimal numbers.
 2. Add the exponents of the powers of 10.
 3. Place the new power of 10 with the decimal in scientific notation form.
 4. If decimal number is greater than 10, count the number of times the decimal moves to the left, and add this number to the exponent.
- To **divide** numbers in scientific notations:
 1. Divide the decimal numbers.
 2. Subtract the exponents of the powers of 10.
 3. Place the new power of 10 with the decimal in scientific notation form.
 4. If the resulting decimal number is less than 1, move the decimal point to the right and decrease the exponent by the number of places that the decimal point moved.

REVISE LESSONS 12 - 17. THEN DO TOPIC 4 TEST 1 IN ASSIGNMENT BOOK 3.

ANSWERS TO PRACTICE EXERCISES 12 - 17.

Practice Exercise 12

1. (a) 5^2 (b) 3^6 (c) 6^2x^5 (d) m^5n^3 (e) $4^3a^4b^3c^2d$
 2. (a) 4 (b) 729 (c) 64 (d) 1 (e) 14 400
 3. (a) $12a^6$ (b) $18x^9y^6$ (c) $48p^4q^5$
(d) $42a^4b^6$ (e) $32m^7n^{10}$
 4. (a) 16 (b) a^4 (c) $81x$
(d) $9x^2y^5$ (e) $3a^5$ (f) $\frac{64m^4}{n}$
 5. (a) x^{15} (b) $4x^6$ (c) m^3n^{12}
(d) $9x^4y^6$ (e) $16p^{16}q^8$
-

Practice Exercise 13

1. (a) 1 (b) 2 (c) 1 (d) 6 (e) 16
 2. (a) $\frac{1}{512}$ (b) $\frac{1}{243}$ (c) $\frac{1}{125}$
(d) $\frac{1}{343}$ (e) $\frac{1}{512}$
 3. (a) $\frac{1}{s^7}$ (b) $\frac{5}{x^3}$ (c) $\frac{1}{r^6}$
(d) $\frac{1}{b^2c^4}$ (e) $\frac{1}{m^7n^4}$ (f) $\frac{1}{64e^7f^4}$
-

Practice Exercise 14

1. a. $5^{\frac{1}{2}}$ b. $7^{\frac{1}{2}}$ c. $x^{\frac{1}{3}}$
d. $m^{\frac{1}{4}}$ e. $c^{\frac{3}{4}}$
2. a. $\sqrt{8}$ b. $\sqrt[3]{5}$ c. $\sqrt[5]{y^3}$
d. $\sqrt[3]{x^2}$ e. $\sqrt[n]{b^m}$

3. a. $\frac{1}{32}$ b. 8 c. 5
d. 81 e. 9
-

Practice Exercise 15

1. (a) 9.48×10^4 (e) 9.6782×10^3
(b) 7×10^3 (f) 7.564005×10^6
(c) 6×10^7 (g) 5.14509×10^5
(d) 5.44696404×10^8
2. (a) 8.75×10^{-4} (d) 5.96×10^{-2}
(b) 6.6×10^{-7} (e) 8.47×10^{-10}
(c) 4×10^{-3}
3. 7.215×10^9 4. 1×10^{-6}
5. $1.67262178 \times 10^{-27}$ 6. 144 000 000 km
7. (a) 1700 000 (d) 0.0000074
(b) 9900 (e) 0.00090033
(c) 102 000
-

Practice Exercise 16

1. 3.8×10^4 6. 8.115×10^8
2. 6.48×10^5 7. 9.6649×10^9
3. 8.39×10^3 8. 8.239×10^{-5}
4. 2.01×10^{-7} 9. 4.5998×10^2
5. 6.1704×10^7 10. 3.4826×10^2

Practice Exercise 17

1. 1.004×10^{-5} 6. 8.659×10^4
2. 1.62×10^{-4} 7. 1.10193×10^8
3. 9.28×10^{-11} 8. 8.78×10^2
4. 3.4104×10^8 9. 5.132×10^3
5. 4.757×10^{-10} 10. 1.12×10^2
-

END OF TOPIC 3

TOPIC 4

EQUATIONS

- | | |
|-------------------|--|
| Lesson 18: | Simple Equations |
| Lesson 19: | Equations involving Brackets |
| Lesson 20: | Equations Involving Fractions |
| Lesson 21: | Problems Involving Equations |
| Lesson 22: | Formulae and Substitution |
| Lesson 23: | Changing the Subject of the Formula or Equation |

TOPIC 4: EQUATIONS

Introduction



In Mathematics, an **equation** is an equality containing one or more variables. Solving the equation consists of determining which values of the variables make the equality true. In this situation, variables are also known as **unknowns** and the values which satisfy the equality are known as **solutions**. An equation differs from an identity in that it is not necessarily true for all possible values of the variable.

There are many types of equations, and they are found in all areas of mathematics; the techniques used to examine them differ according to their type.

In this topic, you will revise the methods you learnt in Grade 9 to solve simple equations. Then you will learn further equation solving methods. You will also learn mathematical formulas.

You will then look at the methods required to solve equations involving brackets and parentheses. You will also learn how to solve equations involving fractions.

You will learn to solve different types of word problems involving equations.

You will also revise the meaning of formula and study again how to change the subject of a formula.

Lesson 18: Simple Equations



You have learned something about solving simple equations in your Grade 8 and 9 Mathematics.



In this lesson, you will:

- define what inverse means
- identify the two types of inverse
- solve simple equations by using inverse operations.

In Mathematics, inverse operations are operations that 'undo' each other. Most operations have an inverse.

The word “inverse” simply means “the opposite”. It comes from the Latin word “inversus”, which means “to turn upside down or inside out”.

An inverse operation is an operation that undoes what was done by the previous operation.

The four main mathematical operations and their inverses are listed on the table below.

Operation	Inverse Operation
Addition	Subtraction
Subtraction	Addition
Multiplication	Division
Division	Multiplication

There are two types of inverse. One is associated with the operations of addition and subtraction, and the other is associated with the operations of multiplication and division.

Let us look at some examples to show how inversion works.

Addition/Subtraction

Take this simple addition problem.

$$4 + 5 = 9$$

If we want to reverse the addition we just subtract

$$9 - 5 = 4$$

And we are back to where we started.

So the inverse of adding 5 is to subtract 5 and the inverse of subtracting 8 is to add 8.

The same is true for multiplication and division.

Take this example of multiplication problem.

$$3 \times 9 = 27$$

If we want to reverse the multiplication we just divide.

$$27 \div 9 = 3$$

And we are back to where we started.

So, the inverse of multiplying by 9 is to divide by 9 and the inverse of dividing by -4 is to multiply by -4.

These are very simple examples, but holds true for even complex addition, subtraction, multiplication and division.

There are four properties that deal with inverses.

1. The Additive Inverse Property

The additive inverse property states that when you add a number to its opposite, the result is always zero (0).

Examples: (a) $3 + (-3) = 0$ (The additive inverse of 3 is -3.)
(b) $342 + (-342) = 0$ (The additive inverse of 324 is - 342.)

2. The Multiplicative Inverse Property

The multiplicative inverse property states that when you multiply any number by its opposite, the result is always one (1).

Examples: (a) $3 \times \frac{1}{3} = 1$ (The multiplicative inverse of 3 is $\frac{1}{3}$)
(b) $153 \times \frac{1}{153} = 1$ (the multiplicative inverse of 153 is $\frac{1}{153}$)

3. The Additive Property

The additive property states that when you add any number to zero, the result is the same number.

Examples: (a) $3 + 0 = 3$
(b) $342 + 0 = 342$

4. The Multiplicative Property

The multiplicative property states that any time you multiply a number by 1, the number does not change.

Examples: (a) $3 \times 1 = 3$
 (b) $342 \times 1 = 342$

Inverse operations can be used to solve equations.

Let us look at solving equation using the inverse operations for addition and subtraction.

Example 1 Suppose we have the equation $x - 8 = 12$

To solve the equation for x , we must isolate x on the left hand side of the equation. To do this, we need to remove the number 8 which is subtracted from x . This is done by adding the additive inverse of -8 , which is $+8$, so that the sum of this pair of numbers will be zero, thus removing the -8 from the L.H.S. of the equation.

Of course we must add the additive inverse -8 , which is $+8$, to both sides of the equation. Thus we have:

$$\begin{aligned} x - 8 + 8 &= 12 + 8 \\ x + 0 &= 12 + 8 \\ x &= 20 \end{aligned}$$

Example 2

Solve the equation: $x + 9 = 15$

Solution: Add the additive inverse of $+9$ which is -9 to both sides of the equation.

$$\begin{array}{ll} x + 9 + -9 = 15 + -9 & \text{Line 1} \\ x + 0 = 15 - 9 & \text{Line 2} \\ x = 6 & \end{array}$$

Since the inverse has the opposite sign to the number we wish to remove, we normally just write this number with opposite sign on the other side of the equation.

So instead of writing $15 + -9$ as in Line 1 in the example above, we just write $15 - 9$.

Similarly, we do not bother writing the number and its inverse on the left hand side of the equation containing x . (as $+9 + -9$ in Line 1 of the above example).

Neither do we bother writing the sum of 0. (as $+0$ in Line 2 of the above example.)

So the setting out we actually use to solve the equation $x + 9 = 15$ is shown below.

$$\begin{aligned} x + 9 &= 15 \\ x &= 15 - 9 \\ x &= 6 \end{aligned}$$

So adding the additive inverse of a number is equivalent to moving the number to the other side of the equation and changing its sign.

This method is called **transposition**. This is the method we will continue to use when the operations are addition and subtraction.

Here are some more examples.

Example 3

Solve the equation: $y - 12 = 25$

Solution: $y - 12 = 25$

$$y = 25 + 12$$

$$y = 37$$

Move -12 to the right side and then change the sign to +12.

Example 4

Solve the equation: $m + 11 = 16$

Solution: $m + 11 = 16$

$$m = 16 - 11$$

$$m = 5$$

Move +11 to the right side and then change the sign to -11.

Now let us look at solving equation using the inverse operations for multiplication and division.

If the variable we wish to find (say x) is multiplied by a number in the original equation, for example $5x$, then we need to multiply $5x$ by the **multiplicative** inverse of the number in front of x that is 5, so we multiply by $\frac{1}{5}$.

This is done so that the 5 in front of x cancels to 1 because the product of a number and its multiplicative inverse is 1, thus reducing $5x$ to just x .

So if we have $5x$: multiply $5x$ by the multiplicative inverse of 5, which is $\frac{1}{5}$:

$$5 \left(\frac{1}{5} x \right)$$

$$= 1 \cdot x \quad \text{Since } 5 \left(\frac{1}{5} \right) = \frac{5}{5} = 1$$

$$= x$$

Similarly suppose another equation contains the term $5x$.

To reduce $5x$ to x , we multiply it by the inverse of 5, that is $\frac{1}{5}$.

$$\left(\frac{1}{5} \cdot 5x \right) = 1 \cdot x = x$$

We must multiply **both sides** of the equation by the inverse.

Here is an example.

Solve the equation: $8x = 24$

Solution: $8x = 24$

$$\frac{1}{8} \cdot 8x = \frac{1}{8} (24) \longleftarrow \text{Multiply both sides by the inverse of 8, that is } \frac{1}{8}$$

$$1x = \frac{24}{8}$$

Therefore, $x = 3$

To save time in setting out our working we normally just divide both sides of the equation by the number in front of x , since multiplying by the multiplicative inverse of this number is exactly the same as dividing the number by itself, because the result will still be 1.

Here is another example.

Example 2

Solve the equation: $4x = 32$

Solution: $4x = 32$

$$\frac{4x}{4} = \frac{32}{4} \longleftarrow \text{Divide both sides by the number in front of } x, \text{ so divide both sides by 4.}$$

$$1x = \frac{32}{4}$$

$$\therefore x = 8$$

We also multiply by the multiplicative inverse if the unknown variable, say x , in the original equation is **divided** by a number, for example; $\frac{x}{5}$ (which of course is the same as multiplying x by the fraction $\frac{1}{5}$, that is $\frac{1}{5}x$.)

Again, we will need to remove the denominator 5 so that we are left with just x .

Now since $\frac{x}{5} = \frac{1}{5}x$ we can remove the $\frac{1}{5}$, just leaving x , if we multiply $\frac{1}{5}$ by its inverse, 5, thus giving a product of 1.

$$\text{That is: } 5 \left(\frac{1}{5}x \right) = \frac{5}{5}x = 1x = x.$$

We must of course multiply **both sides** of the equation by the multiplicative inverse number.

Here is an example showing the method:

Example 3

Solve the equation: $\frac{x}{9} = 3$

Solution $\frac{x}{9} = 3$

$$\frac{1}{9}x = 3$$

$$9\left(\frac{1}{9}x\right) = 9(3) \quad \leftarrow \text{Multiply both sides by the number in front of } x, \text{ so multiply both sides by } 9.$$

$$1x = 27$$

Therefore, $x = 27$

To save time in setting out we just multiply both sides of the equation by the number which is divided into x , since this number (or denominator) is the inverse, for example the inverse of $\frac{1}{9}$ is **9**.

Example 4

Solve the equation: $\frac{x}{10} = 5$

Solution: $\frac{x}{10} = 5$

$$10\left(\frac{x}{10}\right) = 10(5) \quad \leftarrow \text{Multiply both sides by the number in front of } x, \text{ so multiply both sides by } 10.$$

$$1x = 50$$

Therefore, $x = 50$

Example 5

Solve the equation: $-\frac{x}{6} = 5$

Solution: $-\frac{x}{6} = 5$

$$\frac{x}{-6} = 5 \quad \leftarrow \text{Rewrite } -\frac{x}{6} \text{ with minus sign in the denominator}$$

$$-6\left(\frac{x}{-6}\right) = 5(-6) \quad \leftarrow \text{Multiply both sides by the denominator of } x, \text{ so multiply both sides by } -6.$$

$$1x = -30$$

Therefore, $x = -30$

Below are examples of solving two step equations.

Here is an example.

Example 6

Solve the equation: $2x + 3 = 17$

Solution:

$$2x + 3 = 17$$

$$2x = 17 - 3$$

Move +3 to the R.H.S. of the equation and then change its sign -3. So 2x is isolated on the L.H.S. of the equation.

$$2x = 14$$

$$\frac{2x}{2} = \frac{14}{2}$$

Divide both sides of the equation by 2, so that x is left by itself on the L.H.S. of the equation.

Therefore, $x = 7$

Example 7

Solve the equation: $12x - 4 = 8x - 20$

Solution:

$$12x - 4 = 8x - 20$$

$$12x = 8x - 20 + 4$$

Move -4 to the R.H.S. of the equation and then change its sign +4. So 12x is isolated on the L.H.S. of the equation.

$$12x = 8x - 16$$

$$12x - 8x = -16$$

Move +8x to the L.H.S. of the equation and then change its sign -8x.

$$4x = -16$$

$$\frac{4x}{4} = \frac{-16}{4}$$

Divide both sides of the equation by 4, so that x is left by itself on the L.H.S. of the equation.

$$x = -4$$

It is always advisable to check if your answer is correct. This is done by substituting the value of the variable x into the original equation, solve the equation.

If the value on the L.H.S. equals the value on the R.H.S., then the answer is correct. If not, then you have made a mistake.

For example, to check if the answer $x = -4$ in the above example is correct, we substitute the value of x in the original equation $12x - 4 = 8x - 20$.

$$\begin{aligned} \text{So, if } x = -4, \quad & 12(-4) - 4 = 8(-4) - 20 \\ & -48 - 4 = -32 - 20 \\ & -52 = -52 \end{aligned}$$

Since the L.H.S. = R.H.S., then $x = -4$ is correct.

The goal in solving an equation is to get the variable by itself on one side of the equation and a number on the other side of the equation.

To isolate the variable, we must reverse the operations acting on the variable. We do this by performing the inverse of each operation on both sides of the equation. Performing the same operation on both sides of an equation does not change the validity of the equation, or the value of the variable that satisfies it.

Here are some more examples.

1. Solve: $-6x = 30$

Solution: $-6x = 30$

$$\frac{-6x}{-6} = \frac{30}{-6}$$

Therefore, $x = -5$

Answer: $x = -5$

2. Solve: $\frac{x}{11} = 8$

Solution: $\frac{x}{11} = 8$

$$11\left(\frac{x}{11}\right) = 11(8)$$

Therefore, $x = 88$

Answer: $x = 88$

3. Solve: $\frac{3}{5}x = -2$

Solution: $\frac{3}{5}x = -2$

$$5\left(\frac{3}{5}x\right) = 5(-2)$$

$$3x = -10$$

$$\frac{3x}{3} = \frac{-10}{3}$$

$$x = \frac{-10}{3}$$

Therefore, $x = -3\frac{1}{3}$

Answer: $x = -3\frac{1}{3}$

4. Solve $\frac{x}{4} - 9 = 27$

Solution: $\frac{x}{4} - 9 = 27$

$$\frac{x}{4} = 27 + 9$$

$$\frac{x}{4} = 36$$

$$4\left(\frac{x}{4}\right) = 4(36)$$

Therefore, $x = 144$

Answer: $x = 144$

5. Solve: $3x = -5x + 24$

Solution: $3x = -5x + 24$

$$3x + 5x = 24$$

$$8x = 24$$

$$\frac{8x}{8} = \frac{24}{8}$$

Therefore, $x = 3$

Answer: $x = 3$

6. Solve: $5p + 4 = 3p - 0.5$

Solution: $5p + 4 = 3p - 0.5$

$$5p = 3p - 0.5 - 4$$

$$5p = 3p - 4.5$$

$$5p - 3p = -4.5$$

$$2p = -4.5$$

$$\frac{2p}{2} = -\frac{4.5}{2}$$

Therefore, $p = -2.25$

Answer: $p = -2.25$

NOW DO PRACTICE EXERCISE 18

**Practice Exercise 18**

Solve the following equations:

(a) $x - 5 = 2$

(b) $x - 12 = 20$

(c) $p + 4 = -2$

Answer: _____

Answer: _____

Answer: _____

(d) $q - 7 = -3$

(e) $r + \frac{1}{2} = 6$

(f) $6x = 18$

Answer: _____

Answer: _____

Answer: _____

(g) $-3x = 9$

(h) $2x = 1$

(i) $\frac{1}{7}a = -1$

Answer: _____

Answer: _____

Answer: _____

(j) $-\frac{b}{5} = 2\frac{1}{2}$

(k) $\frac{2}{3}x = 4$

(l) $-\frac{3}{11}a = 6$

Answer: _____

Answer: _____

Answer: _____

(m) $4x + 3 = 11$

(n) $3x + 12 = 36$

(o) $5x - 7 = 13$

Answer: _____

Answer: _____

Answer: _____

(p) $-2p + 5 = -3$

(q) $4x = 3x + 13$

(r) $7x = 2x + 10$

Answer: _____

Answer: _____

Answer: _____

(s) $5x = -7x - 1$

(t) $8x - 5 = 7x + 4$

(u) $9x - 8 = 3x + 4$

Answer: _____

Answer: _____

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 19: Equations Involving Grouping Symbols



You have learned to solve simple equations by using inverses in the previous lesson.

In this lesson, you will:

- solve equations which contain pair of brackets
- solve equations which contain more than one pair of brackets.

Sometimes we are given equation which involves **grouping symbols** such as brackets, parentheses and braces.

For example, consider the following equation which contains a pair of brackets.

$$5(4x - 3) = 25$$

To solve this equation, we must first remove the brackets by expanding them. This is done by multiplying each term in the bracket by the number (or term) immediately to the left of the bracket as what you have learned in Topic 1 of this unit.

Then, we apply the methods you learnt in the previous lesson to find the final solution of the equation. So to solve the equation above, we have the following;

$$5(4x - 3) = 25$$

Start by multiplying out the brackets

$$5(4x) + 5(-3) = 25$$

$$20x - 15 = 25$$

Now, move -15 from the L.H.S. of the equation to the R.H.S.

$$20x = 25 + 15$$

$$20x = 40$$

Now, divide both sides by 20

$$\frac{20x}{20} = \frac{40}{20}$$

Therefore, $x = 2$

As was mentioned in the previous lesson, it is always advisable to check if your answer is correct. This is done by substituting the value of the variable x into the original equation then, solves the equation. If the value on the left side of the equation equals the value on the right side, then the answer is correct. If not, then you have made a mistake.

In the example on the previous page, we found that the answer to the equation $5(4x - 3) = 25$ was $x = 2$. To check if the answer $x = 2$ is correct, we substitute the value of x in the original equation.

$$\begin{aligned}\text{Hence,} \quad & 5(4x - 3) = 25 \\ & 5[4(2) - 3] = 25 \\ & 5(8 - 3) = 25 \\ & 5(5) = 25 \\ & 25 = 25\end{aligned}$$

Now look at the following example.

Example 2

$$\text{Solve } 5(x + 4) + 3(4x + 3) = 63$$

This equation contains more than one pair of grouping symbols. To solve the equation, we first expand every pair of grouping symbols in the equation.

$$\begin{aligned}\text{Hence, the solution is:} \quad & 5(x + 4) + 3(4x + 3) = 63 \\ & 5(x) + 5(4) + 3(4x) + 3(3) = 63 && \text{Expand} \\ & 5x + 20 + 12x + 9 = 63 \\ & 17x + 29 = 63 && \text{Collect like terms} \\ & 17x = 63 - 29 && \text{Transposition} \\ & 17x = 34 \\ & \frac{17x}{17} = \frac{34}{17} && \text{Divide both sides by 17} \\ & x = 2\end{aligned}$$

Therefore, the solution is $x = 2$.

Check: when $x = 2$

$$\begin{aligned}5(2 + 4) + 3[(4)(2) + 3] &= 63 \\ 5(6) + 3(8 + 3) &= 63 \\ 30 + 3(11) &= 63 \\ 30 + 33 &= 63 \\ 63 &= 63\end{aligned}$$

Sometimes we are asked to solve equations with grouping symbols on both sides of the equal sign.

Example 3

Solve $7 + 4(5x - 2) - 2(3x - 2) = 5(x + 6)$

Solution: $7 + 4(5x - 2) - 2(3x - 2) = 5(x + 6)$

$$7 + 20x - 8 - 6x + 4 = 5x + 30 \quad \text{Expand}$$

$$7 - 8 + 4 + 20x - 6x = 5x + 30$$

$$3 + 14x = 5x + 30 \quad \text{Collect like terms}$$

$$14x - 5x = 30 - 3$$

Move +3 to the right side and then change the sign to -3.

$$9x = 27$$

$$\frac{9x}{9} = \frac{27}{9}$$

Divide both sides by 9

$$x = 3$$

Check: when $x = 3$

$$7 + 4[5(3) - 2] - 2[3(3) - 2] = 5(3 + 6)$$

$$7 + 4(15 - 2) - 2(9 - 2) = 15 + 30$$

$$7 + 4(13) - 2(7) = 15 + 30$$

$$7 + 52 - 14 = 15 + 30$$

$$45 = 45$$

Here is another example.

Example 4

Solve $5(a - 2) + 4(2a + 1) = 6(a + 6)$

Solution: $5(a - 2) + 4(2a + 1) = 6(a + 6)$

$$5a - 10 + 8a + 4 = 6a + 36 \quad \text{Expand}$$

$$13a - 6 = 6a + 36 \quad \text{Collect like terms}$$

$$13a = 6a + 36 + 6$$

$$13a = 6a + 42$$

$$13a - 6a = 42$$

$$7a = 42$$

$$\frac{7a}{7} = \frac{42}{7}$$

$$a = 6$$

Move -6 to the right side and then change the sign to +6.

Move +6a to the left side and then change the sign to -6a.

Divide both sides by 7

Check: when $a = 6$

$$\begin{aligned} 5(a - 2) + 4(2a + 1) &= 6(a + 6) \\ 5(6 - 2) + 4[2(6) + 1] &= 6(6 + 6) \\ 5(4) + 4(12 + 1) &= 6(12) \\ 20 + 4(13) &= 72 \\ 20 + 52 &= 72 \\ 72 &= 72 \end{aligned}$$

Example 5

Solve $4(y - 3) + 3(3y + 1) = 5(y + 3)$

Solution:

$$\begin{aligned} 4(y - 3) + 3(3y + 1) &= 5(y + 3) \\ 4y - 12 + 9y + 3 &= 5y + 15 \\ 13y - 9 &= 5y + 15 \\ 13y &= 5y + 15 + 9 \\ 13y &= 5y + 24 \\ 13y - 5y &= 24 \\ 8y &= 24 \\ \frac{8y}{8} &= \frac{24}{8} \\ y &= 3 \end{aligned}$$

Expand

Collect like terms

Move -9 to the right side and then change the sign to +9.

Move 5y to the left side and then change the sign to -5y.

Divide both sides by 8

Check:

$$\begin{aligned} 4(y - 3) + 3(3y + 1) &= 5(y + 3) \\ 4(3 - 3) + 3[3(3) + 1] &= 5(3 + 3) \\ 4(0) + 3(9 + 1) &= 5(6) \\ 0 + 3(10) &= 30 \\ 30 &= 30 \end{aligned}$$

NOW DO PRACTICE EXERCISE 19

**Practice Exercise 19**

1. Solve the following equations:

(a) $3(x + 1) = 9$

Answer: _____

(b) $3(w - 2) = -12$

Answer: _____

(c) $5(d + 9) = 47$

Answer: _____

(d) $4(m + 2) + 3(m + 1) = 39$

Answer: _____

(e) $3 + 5(5x + 4) - (x + 1) = -26$

Answer: _____

(f) $6(2x - 5) + 2(x - 7) = -44$

Answer: _____

(g) $2(a + 4) + 3(a + 5) = 4(a + 2) + 10$

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.
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Lesson 20: Equations Involving Fractions



You have learned to solve equations involving grouping symbols in the previous lesson.



In this lesson, you will:

- identify the steps in solving equations which contain more than one fraction
- solve equations which contain more than one fraction.

You will remember that in Lesson 18, we solved equations involving fraction by multiplying both sides of the equation by the denominator of the fraction.

For example: Solve $\frac{x}{9} = 4$

$$\frac{x}{9} = 4$$

$$9\left(\frac{x}{9}\right) = 9(4)$$

$$x = 36$$

Multiply both sides by 9

To solve an equation containing two or more fractions, we transform them into equations without fractions. To do this, **get rid of the denominators**. “Getting rid” of the denominators is actually changing them all to the value of 1, which of course does not need to be written, leaving us with a denominator-free equation. We call this technique **clearing of fractions**.

- Step 1: Start by choosing the lowest common denominator (LCD) of the fractions.
 Step 2: Multiply both sides of the equation (every term) by the LCD of the fractions.
 Step 3: Reduce each term to form an equation without fractions.

Example 1 Solve $\frac{2x}{5} + 1 = \frac{13}{5}$

Solution: Clear the fractions as follows:

The LCD is 5. Therefore, multiply every term on both sides of the equation by 5.

Hence, $5\left(\frac{2x}{5} + 1\right) = 5\left(\frac{13}{5}\right)$

$$2x + 5 = 13$$

$$2x = 13 - 5$$

$$2x = 8$$

$$x = 4$$

Check: when $x = 4$

$$\frac{2x}{5} + 1 = \frac{13}{5}$$

$$\frac{2(4)}{5} + 1 = \frac{13}{5}$$

$$\frac{8}{5} + 1 = \frac{13}{5}$$

$$\frac{8+5}{5} = \frac{13}{5}$$

$$\frac{13}{5} = \frac{13}{5}$$

Example 2

Solve: $\frac{x}{3} + \frac{x-2}{5} = 6$

Solution: Clear of fractions as follows:

The LCD of 3 and 5 is 15. Multiply both sides of the equation (every term) by 15,

Hence, $15\left(\frac{x}{3}\right) + 15\left(\frac{x-2}{5}\right) = 15(6)$

Each denominator will now divide into 15 and we have the equation that has been cleared of fractions:

$$5x + 3(x - 2) = 90$$

Solving the equation: $5x + 3(x - 2) = 90$

$$5x + 3x - 6 = 90$$

$$8x - 6 = 90$$

$$8x = 90 + 6$$

$$\frac{8x}{8} = \frac{96}{8}$$

$$x = 12$$

Expand

Collect like terms

Add 6 to both sides

Divide both sides by 8

Check: when $x = 12$

$$\frac{x}{3} + \frac{x-2}{5} = 6$$

$$\frac{12}{3} + \frac{12-2}{5} = 6$$

$$4 + \frac{10}{5} = 6$$

$$4 + 2 = 6$$

$$6 = 6$$

Example 3

Solve: $\frac{x}{2} - \frac{5x}{6} = \frac{1}{9}$

Solution: The LCD of the denominators 2, 6, 9 is 18. Multiply both sides of the equation (every term) by 18 and cancel.

Hence, $18\left(\frac{x}{2}\right) - 18\left(\frac{5x}{6}\right) = 18\left(\frac{1}{9}\right)$

Each denominator will now divide into 18 and we have the equation that has been cleared of fractions:

$$9x - 15x = 2$$

Solve the equation:

$$9x - 15x = 2$$

$$-6x = 2$$

$$\frac{-6x}{-6} = \frac{2}{-6}$$

$$x = -\frac{1}{3}$$

Collect like terms

Divide both sides by -6

Check: when $x = -\frac{1}{3}$

$$\frac{x}{2} - \frac{5x}{6} = \frac{1}{9}$$

$$\frac{-\frac{1}{3}}{2} - \frac{5\left(-\frac{1}{3}\right)}{6} = \frac{1}{9}$$

$$\begin{aligned} -\frac{1}{3}\left(\frac{1}{2}\right) + \left(\frac{5}{3}\right)\left(\frac{1}{6}\right) &= \frac{1}{9} \\ -\frac{1}{6} + \frac{5}{18} &= \frac{1}{9} \\ -\frac{3}{18} + \frac{5}{18} &= \frac{1}{9} \\ \frac{2}{18} &= \frac{1}{9} \\ \frac{1}{9} &= \frac{1}{9} \end{aligned}$$

We see that when a single fraction is equal to a single fraction, then the equation can be cleared by "cross multiplying".

If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.
--

Example 4

Solve: $\frac{x-6}{2} = \frac{x+3}{5}$

Solution: $\frac{x-6}{2} = \frac{x+3}{5}$ we clear the equation by cross multiplying.

$$2(x+3) = 5(x-6)$$

$$5x - 30 = 2x + 6$$

Solve the equation: $5x - 30 = 2x + 6$

$$5x = 2x + 6 + 30$$

Add 30 to both sides

$$5x = 2x + 36$$

$$5x - 2x = 36$$

Add 2x to both sides

$$3x = 36$$

$$\frac{3x}{3} = \frac{36}{3}$$

Divide both sides by 3

$$x = 12$$

Check: when $x = 12$, $\frac{12-6}{2} = \frac{12+3}{5}$

$$\frac{6}{2} = \frac{15}{5}$$

$$3 = 3$$

Example 5

Solve: $\frac{x-3}{x-1} = \frac{x+1}{x+2}$

Solution: $(x-3)(x+2) = (x-1)(x+1)$

Cross multiply

$$x^2 + 2x - 3x - 6 = x^2 - 1$$

Expand

$$x^2 - x - 6 = x^2 - 1$$

$$x^2 - x^2 - x - 6 = -1$$

Subtract x^2 from both sides

$$-x - 6 = -1$$

$$-x = -1 + 6$$

Add 6 to both sides

$$-x = 5$$

Divide both sides by -1

$$x = -5$$

Check: when $x = -5$, $\frac{-5-3}{-5-1} = \frac{-5+1}{-5+2}$

$$\frac{-8}{-6} = \frac{-4}{-3}$$

$$24 = 24$$

Example 6

Solve: $\frac{3x}{4} - \frac{2}{3} = \frac{x}{2} + \frac{5}{6}$

Solution: LCD of

Solution: The LCD of the denominators 2, 3, 4 and 6 is 12. Multiply both sides of the equation (every term) by 12 and cancel.

Hence, $12\left(\frac{3x}{4}\right) - 12\left(\frac{2}{3}\right) = 12\left(\frac{x}{2}\right) + 12\left(\frac{5}{6}\right)$

Each denominator will now divide into 12 and we have the equation that has been cleared of fractions:

$$9x - 8 = 6x + 10$$

Solve the equation: $9x - 8 = 6x + 10$

$$9x - 6x = 10 + 8$$

$$3x = 18$$

$$\frac{3x}{3} = \frac{18}{3}$$

$$x = 6$$

Transposition

Collect like terms

Divide both sides by 3

Check: when $x = 6$

$$\frac{3x}{4} - \frac{2}{3} = \frac{x}{2} + \frac{5}{6}$$

$$\frac{3(6)}{4} - \frac{2}{3} = \frac{6}{2} + \frac{5}{6}$$

$$\frac{18}{4} - \frac{2}{3} = \frac{6}{2} + \frac{5}{6}$$

$$\frac{9}{2} - \frac{2}{3} = \frac{6}{2} + \frac{5}{6}$$

$$\frac{27 - 4}{6} = \frac{18 + 5}{6}$$

$$\frac{23}{6} = \frac{23}{6}$$

NOW DO PRACTICE EXERCISE 20

**Practice Exercise 20**

Solve the following equations:

1. $\frac{x}{2} - \frac{x}{5} = 3$

Answer: _____

2. $\frac{1}{3}x + \frac{1}{4} = \frac{7}{12}$

Answer: _____

3. $\frac{x-2}{5} + \frac{x}{3} = \frac{x}{2}$

Answer: _____

4. $\frac{x-1}{4} = \frac{x}{7}$

Answer: _____

5. $\frac{a}{10} + \frac{3a}{5} = \frac{7}{2}$

Answer: _____

6. $12 - \frac{4x}{3} = \frac{x}{6}$

Answer: _____

7. $\frac{5k}{8} + \frac{k}{12} = \frac{51}{24}$

Answer: _____

8. $\frac{4y}{9} - \frac{2y}{3} = 10$

Answer: _____

9. $\frac{5x+1}{2} + \frac{x-2}{3} = \frac{8x+8}{6}$

Answer: _____

10. $\frac{x+1}{x} = \frac{-7}{x-12}$

Answer: _____

11. $\frac{4y+1}{5} - \frac{y-7}{4} = \frac{2y-5}{10}$

Answer: _____

12. $\frac{8x}{3} - \frac{x-1}{4} = \frac{x+15}{6}$

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 21: Problem Solving Involving Equations



You have learned to solve equations involving fractions in the previous lesson.



In this lesson, you will:

- work out and write equations from a certain word problem
 - solve word problems using equations
 - solve geometric problems using equations.
-

There are real situations where you will use your knowledge of Algebra and solving equations to solve word problems.

“**Word**” equations are problems expressed in words or sentences in which an unknown quantity is to be found.

Many problems which are stated in words have to be translated into mathematical symbols before they can be solved.

We usually use the letter x , though of course we could choose any letter.

The following illustrative examples represent one number in terms of another.

Example 1

Kila is four times as old as Kema. If we let x represent Kema's age, how would you represent Kila's age?

Solution: Kema's age is x .

Since Kila is four times Kema's age, four times x is $4x$.

Therefore, $4x$ represents Kila's age.

Example 2

John has 90 toea in his two pockets. If he has x toea in one pocket, how much has he in the other pocket?

Solution: Let x = the amount in one pocket.

His total amount of money is 90 toea.

Subtracting x toea, in the first pocket, from 90 toea gives us the amount in the other pocket.

Therefore, $90 - x$ is the amount in second pocket.

Using the information given in the problem, we form an algebraic equation containing the unknown number which we represent by a letter.

It is necessary to be able to read word problems with understanding and translate them into algebraic symbols correctly.

The following suggestions will help you to set up equations for solving word problems.

To solve word problems:

1. Read the whole problem very carefully and get a clear understanding of everything that is told in it.
2. Represent the unknown quantity by a letter or symbol.
3. Form an equation expressed in symbols from the stated relations of the problem.
4. Solve the equation.
5. Check with the problem, not with the equation.

Illustrative example expressing a verbal statement as equation:

Twice a certain number increased by 10 equals 30. What is the number?

Solution: Let x represent the number.

$$\underbrace{\text{Twice a certain number}}_{2x} \underbrace{\text{increased by 10}}_{+ 10} \underbrace{\text{equals 30}}_{= 30}$$

Therefore, the equation is written as $2x + 10 = 30$.

We now solve the equation for x .

$$2x + 10 = 30$$

$$2x = 30 - 10 \quad \text{Transposition}$$

$$2x = 20$$

$$\frac{2x}{2} = \frac{20}{2} \quad \text{Divide both sides by 2}$$

$$x = 10$$

Therefore the number is 10.

Check: $2(10) + 10 = 30$

$$20 + 10 = 30$$

$$30 = 30$$

On the next page you will find some more examples.

Example 2

One-third of a number is 6. What is the number?

Solution: Let x represent the number.

Write down the algebraic equation using the given information:

$$\frac{1}{3}x = 6$$

Solve the equation for x :

$$\frac{1}{3}x = 6$$

$$3\left(\frac{1}{3}x\right) = 3(6) \quad \text{Multiply both sides by 3.}$$

$$x = 18$$

Example 3

If sixteen is subtracted from six times a number, the difference is 80. Find the number.

Solution: Let x = the number
 $6x$ = six times the number
 $6x - 16$ = sixteen is subtracted from six times the number
 80 = the given difference

Write the equation: $6x - 16 = 80$

Solve the equation for x : $6x - 16 = 80$

$$6x = 80 + 16 \quad \text{Transposition}$$

$$6x = 96$$

$$\frac{6x}{6} = \frac{96}{6} \quad \text{Divide both sides by 6}$$

$$x = 16$$

Therefore the number is 16.

$$\text{Check: } 6(16) - 16 = 80$$

$$96 - 16 = 80$$

$$80 = 80$$

We can also solve geometric problems using equations.

You will need to be familiar with the geometric properties of triangles, rectangles and quadrilaterals, if you are to understand this part of the lesson.

You have learned these properties in Grade 7, 8 and 9 Mathematics.

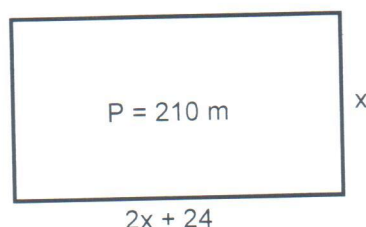
Using the required property, we may form and solve an equation involving the unknown quantity which might be, for example, a length, an area, a perimeter, or the sum of an angle.

We represent the unknown quantity by a letter.

Example 4

A rectangular lot is 24 metres longer than twice its width. The perimeter of the lot is 210 metres. What are the length and width of the lot?

Solution: Representation: Let x = the width
 $2x + 24$ = the length



Using the geometrical fact that the perimeter of a rectangle is twice its length plus twice its width, we can now write the algebraic equation for this problem:

$$2x + 2(2x + 24) = 210$$

Solve the equation for x :

$$2x + 2(2x + 24) = 210$$

$$2x + 4x + 48 = 210$$

$$6x + 48 = 210$$

$$6x = 210 - 48$$

$$6x = 162$$

$$\frac{6x}{6} = \frac{162}{6}$$

$$x = 27 \text{ metres} \quad (\text{width})$$

Now, we find the length: $2x + 24 = 2(27) + 24$

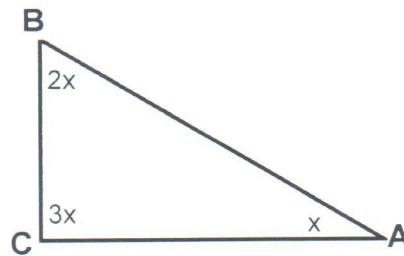
$$= 54 + 24$$

$$= 78 \text{ metres} \quad (\text{length})$$

Example 5

The sum of the three angles of a triangle ABC is 180 degrees. If angle B is twice as large as angle A, and angle C is three times as large as angle A, find the number of degrees in each angle.

Solution: Let x = angle A
 $2x$ = angle B
 $3x$ = angle C
 180° = their sum



Using the given information on the problem, we form the equation:

$$x + 2x + 3x = 180^\circ$$

Solve the equation for x :

$$x + 2x + 3x = 180$$

$$6x = 180$$

$$\frac{6x}{6} = \frac{180}{6}$$

$$x = 30 \quad (\text{angle A})$$

Now we find the other angles:

$$2x = 2(30) = 60 \text{ (angle B)}$$

$$3x = 3(30) = 90 \text{ (angle C)}$$

Therefore, the angles are 30° , 60° and 90° .

NOW DO PRACTICE EXERCISE 21

**Practice Exercise 21**

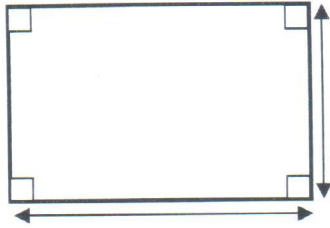
1. Express the following statements as algebraic equations:

- a) Six times a number y is ten times the number x decreased by 95.
 - b) Twice a number x is 24 less than the number increased by 28.
 - c) Two times a number w is 15 less than 34.
 - d) 100 is 12 more than twice a certain number x .
 - e) 7 added to 8 times a certain number x is equal to 12 plus 3 times x .
-

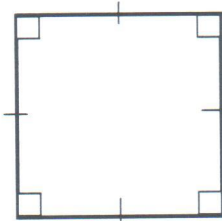
2. Solve the following word problems :

- a) I think of a number, divide by 6, the result is 72. What is the number I am thinking?
- b) The sum of two numbers is 72. The larger number is four more than the smaller number. Find the numbers.
- c) Three angles are supplementary. The second angle is 3 times the first, and the third is 5 times the first. How large is each?

- d) The area of a rectangle is 42 cm^2 . If its width is 3 cm, what is its length?



- e) A square has a perimeter of 32 cm. What is the length of the square?



- f) Let angles A, B, and C are the three angles of a triangle. If angle A is 50° less than angle B, and angle C is 5° more than angle B, how large is each?
- g) Ann's father's age is double her age. The sum of their ages is 60 years. How old is Ann?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 22: Formulae and Substitution



You have learned to solve word problems using equations in the last lesson.



In this lesson, you will:

- recognize different formulae
- substitute a number into a formula
- find a numerical value of the subject of a formula by substitution.

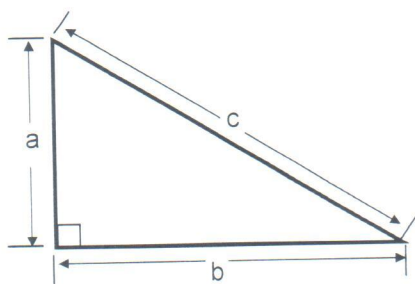
Formulas or formulae (the plural of formula) are mathematical rules, in the form of equations which connect two or more variables. It has a dependent variable on one side and an expression involving independent variable/s on the other side. The variables (letters) often stand for quantities that can be measured for example, length, area, speed, volume, etc.

For example, **A** stands for area, **P** for perimeter, and **V** for volume.

The single variable or letter on the left hand side of the equation or formula, is called the **subject** of the formula.

The following formulas are some of those which you should be familiar with. Though you are not expected to memorize them, it is very important that you know all of them.

a. Right Triangle



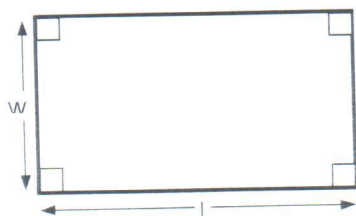
$$c^2 = a^2 + b^2 \quad (\mathbf{c} \text{ is the subject of the formula})$$

$$P = a + b + c \quad (\mathbf{P} \text{ is the subject of the formula})$$

$$A = \frac{1}{2}ab \quad (\mathbf{A} \text{ is the subject of the formula})$$

where **c** is the hypotenuse and **a** and **b** are the legs.

2. Rectangle

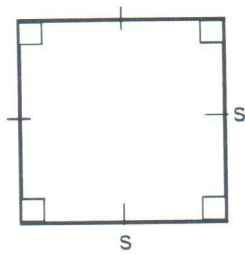


$$P = 2l + 2w \quad (\mathbf{P} \text{ is the subject of the formula})$$

$$A = lw \quad (\mathbf{A} \text{ is the subject of the formula})$$

where **l** is the length and **w** is the width.

3. Square

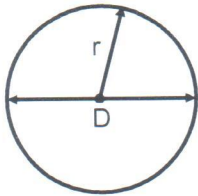


$$P = 4s \quad (P \text{ is the subject of the formula})$$

$$A = s^2 \quad (A \text{ is the subject of the formula})$$

where **s** is the length of the side of the square.

4. Circle



$$C = 2\pi r \text{ or } C = \pi D \quad (C \text{ is the subject of the formula})$$

$$A = \pi r^2 \quad (A \text{ is the subject of the formula})$$

Where **C** is the circumference, **r** is the radius, **D** is the diameter.

5. Investment problems are usually solved by the formula:

$$I = prt \quad (I \text{ is the subject of the formula})$$

where **I** represents the amount of Kina in the interest, **p** represents the amount of kina in the principal, **r** represents the annual rate of interest, and **t** represents the time or number of years the principal is invested.

5. Distance problems are usually solved by the formulae:

$$D = st \quad (d \text{ is the subject of the formula})$$

$$s = \frac{D}{t} \quad (s \text{ is the subject of the formula})$$

where, **D** is the distance travelled and expressed as a number of units of length; such as kilometres (km), metres (m), miles (mi), feet (ft), etc.

s is the speed or rate and expressed as a number of units of length per unit time; such as kilometre per hour, metre per minute, miles per hour, feet per seconds, etc.

and **t** is the time and expressed as a number of units of time; such as seconds, minutes, hours, days etc.

6. The number of toea in Kina:

$$\text{Number of toea} = 100 \times \text{number of Kina}$$



$$t = 100K \quad (t \text{ is the subject of the formula})$$

where, **t** represent the number of toea, and **K** represent the number of Kina

Evaluating Formulas by Substitution

To find the numerical value of the subject of the formula, that is the single value on the left-hand side of the equation, we substitute given numerical value for the variables in the right-hand side of the formula.

Example 1

Consider the formula $C = 6xy$. Find C if $x = 3$ and $y = 4$.

Solution: C is the subject of the formula.

To find its value, we evaluate the expression on the right-hand side of the equal sign, $6xy$, by substituting $x = 3$ and $y = 4$.

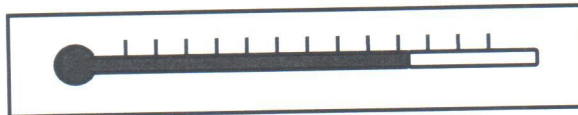
$$\begin{aligned}\text{This gives us} \quad C &= 6xy \\ C &= 6(3)(4) \\ &= 72\end{aligned}$$

Therefore, the value of C is 72.

Example 2

The formula for converting degrees Fahrenheit (**F**) to degrees Celsius (**C**) is:

$$C = \frac{5}{9}(F - 32)$$



What is 122°F in degrees Celsius?

Solution: Substitute $F = 122$ into $C = \frac{5}{9}(F - 32)$.

$$\text{Hence, this gives us } C = \frac{5}{9}(122 - 32)$$

$$= \frac{5}{9} \cancel{(90)}^{10}$$

Cancel

$$= 5(10)$$

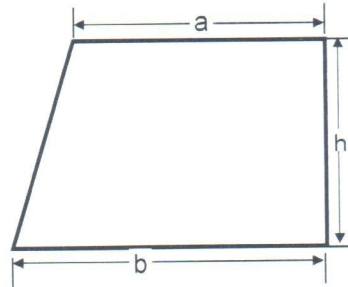
$$= 50$$

Therefore, $122^{\circ}\text{F} = 50^{\circ}\text{C}$.

Example 3

The formula for finding the area of a trapezoid is $A = \frac{h}{2}(a + b)$.

If $a = 6$ cm, $b = 9$ cm and $h = 4$ cm, calculate A .



Solution: Substitute $a = 6$, $b = 9$ and $h = 4$ in the formula: $A = \frac{h}{2}(a + b)$

Hence, $A = \frac{h}{2}(a + b)$

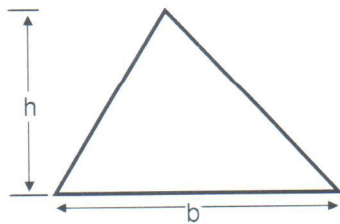
$$A = \frac{4}{2}(6 + 9)$$

$$= 2(15)$$

$$= 30 \text{ cm}^2$$

Example 4

The area of a triangle of base **b** and height **h** is given by the formula: $A = \frac{1}{2}bh$



Find the area of a triangle of base 10 cm and height 8 cm.

Solution: Substitute $b = 10$ and $h = 8$ into: $A = \frac{1}{2}bh$

$$= \frac{1}{2}(10)(8)$$

$$= \frac{1}{2}(80)$$

$$= 40 \text{ cm}^2$$

Therefore, the area of the triangle is 40 cm^2 .

Example 5

The x and y coordinates of points on a curve are connected by the equation below:

$$y = 3x^2 - 4x + 2$$

Calculate y when $x = 4$.

Solution: Substitute $x = 4$ into: $y = 3x^2 - 4x + 2$

$$\begin{aligned}\text{Hence, } y &= 3(4)^2 - 4(4) + 2 \\ &= 3(16) - 16 + 2 \\ &= 48 - 16 + 2 \\ &= 34\end{aligned}$$

Therefore, the value of $y = 34$.

Example 6

Ralph intends to draw a graph showing the population (p) of ants over a period of days (d). The formula he discovered connecting the p and d was: $p = d^2 + 2$

Complete the table of values below, where he will need to draw the graph.

d	1	2	3	4
$p = d^2 + 2$				

- Solution:
- (i) when $d = 1$: $p = d^2 + 2$
 $= (1)^2 + 2$
 $= 1 + 2$
 $= 3$
 - (ii) when $d = 2$: $p = d^2 + 2$
 $= (2)^2 + 2$
 $= 4 + 2$
 $= 6$
 - (iii) when $d = 3$: $p = d^2 + 2$
 $= (3)^2 + 2$
 $= 9 + 2$
 $= 11$
 - (iv) when $d = 4$: $p = d^2 + 2$
 $= (4)^2 + 2$
 $= 16 + 2$
 $= 18$

Answer:

d	1	2	3	4
$p = d^2 + 2$	3	6	11	18

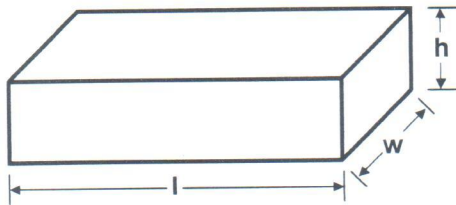
NOW DO PRACTICE EXERCISE 22



Practice Exercise 22

Solve the following:

1. The height (**h**) of a rectangular prism of volume (**V**), length (**l**) and width (**w**) is given by the formula: $h = \frac{V}{lw}$. If $V = 54 \text{ cm}^3$, $l = 9 \text{ cm}$ and $w = 3 \text{ cm}$, find **h**.



Answer: _____

2. Given that the speed (**s**), time (**t**) and distance (**D**) are connected by the formula: $s = \frac{D}{t}$, find the speed of a car which travels 250 km in 5 hours.

Answer: _____

3. The **x** and **y** coordinates of points on a line are connected by the equation: $y = 4x + 3$.

Complete the table of values shown below:

X	-1	0	1	2
$y = 4x + 3$				

Answer: _____

4. If $S = n(a + b)$, find S if $n = 6$, $a = 4$ and $b = 5$.

Answer: _____

5. The average of two numbers, m and n , is given by the formula $A = \frac{m + n}{2}$.
Find A , if:

(a) $m = 6$ and $n = 10$

Answer: _____

(b) $m = 12$ and $n = 18$

Answer: _____

(c) $m = 4.5$ and $n = 3.1$

Answer: _____

6. For the formula $y = mx + b$, find y if:

(a) $m = 2$, $x = 7$ and $b = 3$

Answer: _____

(b) $m = 0$, $x = 3$ and $b = 2$

Answer: _____

(c) $m = \frac{1}{2}$, $x = 4$ and $b = 2$

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 23: Changing the Subject of a Formula



You have learned to find the numerical value of the subject of the formula by substitution in the last lesson.



In this lesson, you will:

- change the subject of a simple formula and more complicated formula.

You came across the meaning of the subject of a formula at the beginning of Lesson 22. It is a variable which is expressed in terms of other variables involved in the formula.

Formulas are written so that a single variable, the subject of the formula is on the left-hand side (L.H.S.) of the equation. Everything else goes on the right-hand side of the equation. We evaluate the formula by substituting for the variables on the right-hand side.

For example, in the formula $P = 2l + 2w$, the subject of the formula is P .

To find P , in the example, we substitute the values l and w in the R.H.S. of the equation.

Similarly, the subject in the formula: $v = u + at$ is v .

Changing the subject of a formula

To change the subject of a formula, begin with the variable or letter to become the new subject.

The volume of a rectangular prism or cuboid is the product of its length, width and height. So the formula connecting the volume V of the prism with its length l , width w and height h is:

$$V = lwh \quad V \text{ is the subject of this formula.}$$

Suppose we want to change the subject of this formula to be the length l .

To do this, rearrange the formula so that the length is the subject by swapping the L.H.S. and the R.H.S. so that the group of terms containing the required new subject is on the left-hand side of the equation.

So swapping sides of $V = lwh$, we have now $lwh = V$.

Now we remove w and h on the left-hand side of this equation by dividing both sides of the equation by wh .

$$\frac{lwh}{wh} = \frac{V}{wh}$$

Therefore,
$$l = \frac{V}{wh}$$

Example 2

Given the formula in $v = u + at$.

- (a) Make "**u**" the subject of the formula.
- (b) Make "**a**" the subject of the formula.
- (c) Make "**t**" the subject of the formula.

Solution:

Rearrange the formula: $v = u + at$

(a) Swap sides $u + at = v$
 $u + at - at = v - at$ subtract **at** from both sides
 $u = v - at$

(b) To find **a**, form (a): $u - u + at = v - u$ subtract **u** from both sides
 $at = v - u$
 $\frac{at}{t} = \frac{v - u}{t}$ divide both sides by **t**

Therefore, $a = \frac{v - u}{t}$

(c) To find **t**, form (a): $u - u + at = v - u$ subtract **u** from both sides
 $at = v - u$
 $\frac{at}{a} = \frac{v - u}{a}$ divide both sides by **a**

Therefore, $t = \frac{v - u}{a}$

Example 3

The formula connecting the circumference **C** of a circle and its diameter **d** is $C = \pi d$.
 Make **d** the subject of the formula.

Solution: $C = \pi d$
 $\pi d = C$ swap sides
 $\frac{\pi d}{\pi} = \frac{C}{\pi}$ divide both sides by π
 Therefore, $d = \frac{C}{\pi}$

Example 4

Make **y** the subject of the formula in $x = \frac{y + z}{y - z}$.

Solution:

$$x = \frac{y + z}{y - z}$$

$$x(y - z) = (y + z)$$

$$xy - xz = y + z$$

$$xy - y = z + xz$$

$$y(x - 1) = z(x + 1)$$

$$\frac{y(x - 1)}{(x - 1)} = \frac{z(x + 1)}{(x - 1)}$$

Multiply both sides by $(y - z)$.

Expand the brackets

Rearrange and swap

Factorize

Divide both sides by $x - 1$

Therefore,
$$y = \frac{z(x + 1)}{(x - 1)}$$

Example 5

Make **x** the subject of the formula $y = 6 - 5x$.

Solution:

$$y = 6 - 5x$$

$$y + 5x = 6$$

$$5x = 6 - y$$

$$x = \frac{6 - y}{5}$$

add $5x$ to both sides

subtract y from both sides

divide both sides by 5

More worked-out problems to change the subject of a formula.

Example 6

In the formula $S = a + (n - 1)d$ make **d** the subject of the formula. Then, find the value of **d** when **S** = 10, **a** = 2 and **n** = 5.

Solution:
$$S = a + (n - 1)d$$

$$a + (n - 1)d = S$$

$$(n - 1)d = S - a$$

$$\frac{(n - 1)d}{(n - 1)} = \frac{S - a}{(n - 1)}$$

$$d = \frac{S - a}{(n - 1)}$$

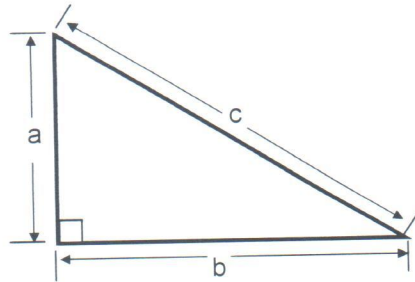
To find the value of d , substitute $S = 10$, $a = 2$ and $n = 5$ with $d = \frac{S - a}{(n - 1)}$.

Hence we have
$$d = \frac{10 - 2}{5 - 1} = \frac{8}{4} = 2$$

Therefore, $d = 2$.

Example 7

For right angled triangle, the square of the hypotenuse (c) is equal to the sum of the squares of its other two sides (a and b).



- Form the formula for the above statement and find out c if $a = 4$ and $b = 3$.
- Make a the subject of the formula and find a if $c = 10$ and $b = 8$.

Solution:

- From the above statement, $c^2 = a^2 + b^2$

When $a = 4$ and $b = 3$,

$$c^2 = 4^2 + 3^2$$

$$= 16 + 9$$

$$c^2 = 25 \text{ or } c = \sqrt{25}$$

Therefore,
$$c = 5$$

- Form (a), change the subject of the formula:

$$a^2 = c^2 - b^2$$

When $c = 10$ and $b = 8$,

$$a^2 = 10^2 - 8^2$$

$$= 100 - 64$$

$$a^2 = 36 \text{ or } a = \sqrt{36}$$

Therefore,
$$a = 6$$

NOW DO PRACTICE EXERCISE 23

**Practice Exercise 23**

1. Make x the subject of each of the following formulae.

(a) $y = 4x$

(b) $y = \frac{x+2}{4}$

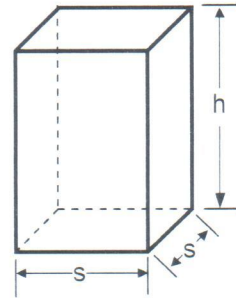
(c) $y = 2x + 3$

(d) $y = abx$

(e) $q = \frac{3(x-4)}{2}$

2. A box with a square base has its volume given by the formula $V = s^2h$ and its surface area given by $A = 2s^2 + 4sh$.

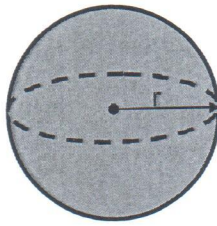
(a) Make h the subject of both formulae.



(b) Find h if $A = 24 \text{ cm}^2$ and $s = 2 \text{ cm}$.

(c) Find h if $V = 250 \text{ cm}^3$ and $s = 10 \text{ cm}$.

3. The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$.



- (a) Rearrange the formula to give r in terms of V .

- (b) Find the value of r , to the nearest centimeters, when $V = 103.04 \text{ cm}^3$.

-
4. The formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ is used to find the focal length of a lens.

- (a) Make v the subject of the formula.

- (b) Find v if $f = 6$ and $u = 10$

5. Raka's pay (**P**) in kina each week depends on the number of sales (**s**) she makes. The formula is **$P = 20(3s + 4)$** .

- (a) How much is Raka paid if she makes no sales?
- (b) How much is she paid if she makes 10 sales?
- (c) How many sales must she make to earn K500 in a week?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.
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TOPIC 4: SUMMARY



This summarizes some of the important ideas and concepts to remember.

- In **solving simple equations**, we apply the properties of inverses.
- If a number and its additive inverse are added, then the sum is equal to zero, for example the additive inverse of $+4$ is -4 .
- **Transposition** means moving the number to the other side of the equation and changing the sign. It is equivalent to adding the additive inverse of a number.
- **The product of a number and its multiplicative inverse is equal to 1**, for example the multiplicative inverse of 2 is $\frac{1}{2}$.
- **To solve equations involving grouping symbols**, remove the grouping symbols by expanding the brackets or parentheses and then applying the properties of inverses.
- **To solve equations involving fractions**, transform them into equations without fractions. To clear the fractions, multiply every term on both sides of the equation by the least common denominators of the fractions, collect like terms and then apply the properties of inverses to solve the resulting equation.
- **To solve word problems:**
 - Read the whole problem very carefully and get a clear understanding of everything that is told in it.
 - Represent the unknown quantity by a letter or symbol.
 - Form an equation expressed in symbols from the stated relations of the problem.
 - Solve the equation.
 - Check with the problem, not with the equation.
- **Formulae or formulas** are mathematical rules in the form of equations which connect two or more variables.
- **Substitution** is the process of replacing the letters or variables in a formula.
- **Subject of a formula** is the single variable or letter on the left-hand side of the equation or formula which is expressed in terms of the other variables on the right-hand side of the equation.
- **To change the subject of a formula**, rearrange the formula by swapping the L.H.S. and the R.H.S. so that the group of terms containing the required new subject is on the left-hand side of the equation.

REVISE LESSONS 18 - 23. THEN DO TOPIC TEST 4 IN ASSIGNMENT BOOK 3.

ANSWERS TO PRACTICE EXERCISES 18-23

Practice Exercise 18

- | | |
|--------------------------|-------------------------|
| (a) $x = 7$ | (l) $a = -22$ |
| (b) $x = 32$ | (m) $x = 2$ |
| (c) $p = -6$ | (n) $x = 8$ |
| (d) $q = 4$ | (o) $x = 4$ |
| (e) $r = 5\frac{1}{2}$ | (p) $p = 4$ |
| (f) $x = 3$ | (q) $x = 13$ |
| (g) $x = -3$ | (r) $x = 2$ |
| (h) $x = \frac{1}{2}$ | (s) $x = -\frac{1}{12}$ |
| (i) $a = -7$ | (t) $x = 9$ |
| (j) $b = -12\frac{1}{2}$ | (u) $x = 2$ |
| (k) $x = 6$ | |

Practice Exercise 19

- | | |
|-----------------------|--------------|
| 1. (a) $x = 2$ | (e) $x = -2$ |
| (b) $w = -2$ | (f) $x = 0$ |
| (c) $d = \frac{2}{5}$ | (g) $a = -5$ |
| (d) $m = 4$ | |

Practice Exercise 20

- | | |
|--|-------------------------|
| 1. $x = 10$ | 7. $k = 3$ |
| 2. $x = 1$ | 8. $y = -45$ |
| 3. $x = 12$ | 9. $x = 1$ |
| 4. $x = \frac{7}{3}$ or $2\frac{1}{3}$ | 10. $x = 6$ or $x = -2$ |
| 5. $a = 5$ | 11. $y = 7$ |
| 6. $x = 8$ | 12. $x = 1$ |

Practice Exercise 21

1.
 - a) $6y = 10x - 95$
 - b) $2x = (x - 24) - 28$
 - c) $2w = 34 - 15$
 - d) $100 = 2x + 12$
 - e) $8x + 7 = 12 + 3x$
2.
 - a) 432
 - b) 34 and 38
 - c) 20° , 60° and 100°
 - d) 14 cm
 - e) 8 cm
 - f) $A = 25^\circ$, $B = 75^\circ$ and $C = 80^\circ$
 - g) 20 years

Practice Exercise 22

1. 2 cm
2. 50 km per h
- 3.

x	-1	0	1	2
$y = 4x + 3$	-1	3	7	11

4. 54
5.

a) 8	b) 15	c) 3.8
------	-------	--------
6.

a) 17	b) 2	c) 4
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Practice Exercise 23

1.
 - (a) $x = \frac{y}{4}$
 - (b) $x = 4y - 2$
 - (c) $x = \frac{y - 3}{2}$
 - (d) $x = \frac{y}{ab}$
 - (e) $x = \frac{2q}{3} + 4$

2. (a) $h = \frac{V}{s}$; $h = \frac{A}{4s} - \frac{s}{2}$

(b) $h = 2 \text{ cm}$

(c) $h = 2.5 \text{ cm}$

3. (a) $r = \sqrt[3]{\frac{3V}{4\pi}}$

(b) $r = 3 \text{ cm}$

4. (a) $v = \frac{fu}{u - f}$

(b) $v = 15$

5. (a) K80

(b) K680

(c) 7 sales

END OF UNIT 3

NOW YOU MUST COMPLETE ASSIGNMENT 3.

REFERENCES

- FODE Grade 10 Formal Mathematics Unit 3, Algebra, 2008 Edition
 - Oxford: Modular Mathematics; Intermediate GCSE for AQA, by: Brian Gaulter and Clive Simpson
 - Pearson: Signpost Mathematics 8, Second Edition by: A. McSeveny, R. Conway and S. Wilkes
 - http://www.mathsteacher.com.au/year10/ch01_algebraic/04_binomials/binom.htm
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 - <http://www.mathsisfun.com/algebra/like-terms.html>
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 - <http://www.algebra-class.com/binomial.html#sthash.QNUvY1wO.dpuf>
 - <http://www.study.maths.co.uk/topics/factorisingQuadraticExpressions.php>
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FODE PROVINCIAL CENTRES CONTACTS

FODE PROVINCIAL CENTRES

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SUBJECT AND GRADE TO STUDY

Grade Levels	Subjects
Grades 7 and 8	1. English
	2. Mathematics
	3. Science
	4. Social Science
	5. Making a Living
	6. Personal Development
	7. English
Grades 9 and 10	1. English
	2. Formal Mathematics
	3. Practical Mathematics
	4. Science
	5. Social Science
	6. Commerce
	7. Design and Technology- Computing
	8. Personal Development
Grades 11 and 12	1. English
	• Applied English
	• Language and Literature
	2. Mathematics
	• Mathematics A
	• Mathematics B
	3. Science
	4. Social Science
	• History
	• Geography
	• Economics
	5. Business Studies
	6. Personal Development
	7. ICT

REMEMBER:

- For Grades 7 and 8, you are required to do all six (6) courses.
- For Grades 9 and 10, you must study English, Mathematics, Science, Personal Development, Social Science and Commerce. Design and Technology-Computing is optional.
- For Grades 11 and 12, you are required to complete seven (7) out of thirteen (13) courses to be certified. Your Provincial Coordinator or Supervisor will give you more information regarding each subject.

Certificate in Matriculation

CORE COURSES

Basic English
English 1
English 2
Basic Maths
Maths 1

Maths 2

History of Science & Technology

OPTIONAL COURSES

Science Streams: Biology

Chemistry, Physics and Social Science Streams:

Geography, Introduction to Economics and Asia and the Modern World

REMEMBER:

You must successfully complete 8 courses; 5 compulsory and 3 optional